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Moduli stabilisation versus chirality for MSSM like type IIB orientifolds

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Abstract: We investigate the general question of implementing a chiral MSSM like D-brane sector in Type IIB orientifold models with complete moduli stabilisation via F-terms induced by fluxes and space-time instantons, respectively gaugino condensates. The prototype examples are the KKLT and the so-called large volume compactifications. We show that the ansatz of first stabilising all moduli via F-terms and then introducing the Standard Model module is misleading, as a chiral sector notoriously influences the structure of non-perturbative effects and induces a D-term potential. Focusing for concreteness on the large volume scenario, we work out the geometry of the swiss-cheese type Calabi-Yau manifold $\mathbb{P}_{[1,3,3,3,5]}[15]_{(3,75)}$ and analyse whether controllable and phenomenologically acceptable Kähler moduli stabilisation can occur by the combination of F- and D-terms.

Keywords: Superstring Vacua, Intersecting branes models, Flux compactifications.

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Contents

1. Introduction

It is the main goal of string phenomenology to find realistic string models which are predictive (in the weak sense[1]) and which, for a fixed background, fix dynamically all low energy parameters. Indeed, it would be an important advance and a proof of principle to find a globally consistent string compactification which contains and combines all the various mechanisms of moduli stabilisation and D-brane model building techniques leading to the MSSM localised on some (intersecting) D-branes and leading to a predictive framework for cosmology. This means it would allow us to precisely compute all MSSM and cosmological parameters from the underlying dynamically stabilised string model.

In recent years some progress has been made towards actually achieving this goal in that new ways of fixing moduli in string compactifications have been found (see [1−8] for

[1]We are not discussing here problems related to the plethora of string models (the string landscape) and its consequences for the predictivity of string theory.

reviews). The most often discussed cases are Type IIB orientifolds, where a combination of background fluxes [9] and non-perturbative terms were argued to allow to fix all closed string moduli [10]. Here the complex structure moduli and the dilaton are stabilised by three-form fluxes and the Kähler moduli by non-perturbative terms arising from D3-brane instantons and/or gaugino condensation on D7-branes (for a systematic analysis see for instance [11 – 15]).[2]

Quite remarkably, generalising the KKLT scenario, phenomenologically appealing models have been found, for which in a non-supersymmetric minimum of the scalar potential the overall volume $\mathcal{V}$ of the Calabi-Yau manifold is fixed at a very large value like for instance $\mathcal{V} \simeq 10^{15}$, where α'-corrections to the Kähler potential compete with non-perturbative contributions to the superpotential [16]. The Kähler moduli, on whose associated cycles the euclidean D3-brane instantons are wrapped, are fixed by non-perturbative contributions to the superpotential at small volume, $\tau \simeq \log(\mathcal{V})$.

These models have been called large volume compactifications and many of their low energy features have been worked out, including a computation of supersymmetry breaking soft terms [17, 18] and its collider signatures [19]. We think it is fair to say that such an investigation has set a new standard for doing string phenomenology.

Let us point out that the main implicit assumption made in the KKLT and large volume scenarios was that it is a valid procedure to split the construction of such string models into two steps. The first one is to fix all moduli by a combination of fluxes and non-perturbative effects. After that has been achieved, the second step is to introduce the *module* of the MSSM on some intersecting respectively magnetised D7-branes. On a phenomenological level this might be a fair attempt. It is the aim of this paper to investigate more closely, whether the conditions appearing in string theory justify such a procedure. For concreteness and because of their attractive phenomenological properties, in this paper we will mainly discuss the large volume scenario but would like to stress that all the structure and constraints we find directly carry over to other constructions including the KKLT scenario [10].

Essentially generalising the arguments of [20 – 23], in this paper we would like to emphasise that there is a fundamental problem when combining a chiral MSSM like module with the moduli stabilisation module. After reviewing the main ingredients of the large volume scenario in section 2, we will point out two general features which a chiral D7-brane sector introduces:

- It leads to the generation of a D-term potential, which appears at lower order in the $1/\mathcal{V}$ expansion of the scalar potential. Therefore, there is the danger of destabilising the large volume minimum.
- On the intersection of the D7-branes with the E3-brane instantons extra charged *chiral* fermionic zero modes can appear, spoiling the generation of an uncharged superpotential.

[2] In the remainder of this paper we will mainly consider D3-brane instantons. However, let us already mention that the results we obtain carry over to gaugino condensation on D7-branes.

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We will argue that, as studied for instance in [21, 22, 24], a non-perturbative superpotential including matter fields does not resolve the second problem, as in our bottom-up approach we do not want to give VEVs to MSSM matter fields. This is in contrast to [21, 25, 22, 24], where these matter field superpotentials were used in the hidden sector for uplifting the AdS minimum to de Sitter. There, no direct phenomenological constraints arise for the VEVs of the hidden sector matter fields.

As we will show, due to the chirality of the D7-brane sector, in our MSSM case the D7-branes and the E3-branes should better wrap in some sense orthogonal four-cycles. This means that not all the sizes of the D7-branes are fixable by instanton induced F-terms. However, it is precisely the sizes of these cycles on which some of the low-energy MSSM parameters depend. Therefore, by this effect we seem to loose part of the very predictive power of the models [17 − 19].

The combination of both aspects mentioned in the previous paragraph provides a natural solution to the problem of fixing all Kähler moduli for an MSSM like model. Non-perturbative effects fix part of the Kähler moduli except some of the ones controlling the size of the MSSM branes. These latter are fixed by the vanishing of the D-term potential. Since it is a D-term, there could also be charged matter contributions. Again, by requiring not to break the MSSM gauge symmetry already at the high scale they should better have vanishing vacuum expectation values. Note that, for the uplifting physics in the hidden sector this argument does not apply.

Furthermore, we find that in the case of multiple contributions to the instanton generated superpotential, there are additional terms potentially destabilising the large volume minimum found in [16, 17] for the single instanton case. However, requiring that the four-cycles the instantons wrap do not intersect leads to the familiar form of the scalar potential [16, 17]. We also allow for more general rigid four-cycles, homologically described by linear combinations of the basic blow-up cycles which can be understood as rigid, singular configurations of the basic ones. In this case, the F-terms also take a slightly different form than described in [16, 17].

In the second part of this paper, we will investigate some of these aspects for the swiss cheese type Calabi-Yau manifold defined via the resolution of the singular hypersurface in a weighted projective space $\mathbb{P}_{[1,3,3,3,5]}[15]_{(3,75)}$.[3] Working out the toric geometry of the resolution, we will show that this space contains rigid four-cycles, on which wrapped E3-instantons have the right zero mode structure to give a contribution to the uncharged superpotential. Introducing also a set of chirally intersecting and magnetised D7-branes, we find that, for vanishing VEVs for charged matter fields, the combination of F- and D-terms fixes the four-cycles at the boundary of the Kähler cone, where however the sizes of the instantons and D7-branes remain finite and of the same scale. For non-vanishing VEVs of some of the matter fields the situation gets even improved.

[3] In appendix B, we will also provide the geometric data for a second new swiss-cheese Calabi-Yau, namely the resolution of $\mathbb{P}_{[1,1,3,10,15]}[30]_{(5,251)}$.

2. Type IIB orientifolds

The best understood moduli stabilisation techniques have been developed for the perturbative Type IIB string, which is mainly related to the fact that turning on background three-form fluxes only mildly changes the background geometry by introducing a warp factor [9]. As we will explain at the example of the large volume models of [16, 17], the task of combining moduli stabilisation with an MSSM type gauge sector is non-trivial and more involved than just combining these two separate modules.

2.1 Large volume scenario

In order to explain our arguments, we will use as a prototype example the large volume scenario (LVS) of closed string moduli stabilisation in Type IIB orientifold models [16]. We consider Type IIB orientifolds of a Calabi-Yau manifold $\mathcal{X}$ with in general O7- and O3-planes. The complex structure moduli U are encoded in the holomorphic three-form Ω_3 of $\mathcal{X}$ and the axio-dilaton field reads

$$S = e^{-\phi} + i\, C_0 \,, \tag{2.1}$$

where ϕ is the dilaton and C_0 is the Ramond-Ramond (R-R) zero-form. These moduli are usually fixed by the Gukov-Vafa-Witten superpotential [26]

$$W_{\rm GVW} = \int_{\mathcal{X}} G_3 \wedge \Omega_3 \,, \tag{2.2}$$

arising from three form flux $G_3 = F_3 + iS\, H_3$ supported on the three-cycles of the Calabi-Yau manifold. As for the KKLT scenario [10], it is assumed that the flux vacuum breaks supersymmetry, where the value of the superpotential in the minimum is denoted by W_0.

Concerning the stabilisation of the Kähler moduli, the tree-level no-scale structure of the flux induced scalar potential is broken by both the leading order perturbative α'-corrections to the tree-level Kähler potential and by E3-brane instanton corrections to the superpotential. The Kähler potential including the α'-corrections reads [27]

$$K = -2\ln\left(\hat{\mathcal{V}} + \frac{\xi}{2\, g_s^{3/2}}\right) - \ln\left(S+\overline{S}\right) - \ln\left(-i\int_{\mathcal{X}} \Omega\wedge\overline{\Omega}\right) \,, \tag{2.3}$$

where g_s denotes the string coupling and we have set the supergravity scale as $M_{\rm Pl} = 1$. The string-frame volume $\mathcal{V}$ of the Calabi-Yau manifold is expressed in the following way

$$\mathcal{V} = \frac{1}{3!}\int_{\mathcal{X}} J\wedge J\wedge J = \frac{1}{6}\,\mathcal{K}_{ijk}\, t^i\, t^j\, t^k \,, \tag{2.4}$$

where we have expanded the Kähler form J in some basis $\{\omega_i\}$ of $H^{1,1}(\mathcal{X},\mathbb{Z})$ as $J = \sum_i t^i\omega_i$. The Einstein-frame volume appearing in (2.3) is denoted by a hat. Furthermore, $\mathcal{K}_{ijk}$ denotes the triple intersection number in the chosen basis. The α'-corrections are encoded in ξ in terms of the Euler number χ of the internal manifold $\mathcal{X}$

$$\xi = -\frac{\zeta(3)\,\chi(\mathcal{X})}{2\,(2\pi)^3} \,. \tag{2.5}$$

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In addition to the perturbative corrections, one also takes into account E3-brane instantons in order to break the no-scale structure of the scalar potential. These instantons generate terms in the superpotential of the following form [28]

$$W_{\rm np} = A(S,U)\, e^{-S_{\rm inst}} = A(S,U)\, e^{-\sum_i a_i T_i}\ . \tag{2.6}$$

The Kähler moduli T_i of Type IIB orientifolds with O3- and O7-planes are a particular combination of the Kähler form J and of the R-R four-form C_4

$$T_i = e^{-\phi}\, \frac{1}{2} \int_{D_i} J\wedge J + i \int_{D_i} C_4 = e^{-\phi}\, \tau_i + i\, \rho_i\ , \tag{2.7}$$

where $\{D_i\} \subset H_4(\mathcal{X},\mathbb{Z})$ with $i = 1,\ldots,h_{1,1}(\mathcal{X})$ forms a basis of four-cycles on the internal manifold. The τ_i denote the volume of D_i and ρ_i is the associated axion. For ease of notation, we will express all geometric quantities like $\mathcal{V}$ and τ in string-frame. However, in the supergravity formulas, they have to appear in Einstein-frame which is achieved by substituting $J \to e^{-\phi/2} J$. As already indicated above, the resulting quantities in this frame will be denoted by a hat, i.e. $\hat{\mathcal{V}}$ and $\hat{\tau}$.

For manifolds where the overall volume $\mathcal{V}$ is controlled by one large four-cycle, it was shown that such compactifications admit an interesting minimum of the resulting scalar potential [16]. More precisely, the minimum arises at exponentially large volumes, if $\mathcal{V}$ can be written as

$$\mathcal{V} \sim \tau_b^{\frac{3}{2}} - \sum_{s=1}^{h_{1,1}-1} \tau_s^{\frac{3}{2}}, \tag{2.8}$$

where τ_b denotes the volume of the big four-cycle and τ_s measure the sizes of (small) holes in this geometry. Thus, these models have a “swiss-cheese” like structure [17, 29, 30].

The standard example primarily studied in the literature is the Calabi-Yau manifold $\mathbb{P}_{[1,1,1,6,9]}$[18] which has a codimension three $\mathbb{Z}_3$-singularity [11, 31, 32]. Its resolution introduces a second Kähler modulus T_s so that the volume becomes

$$\mathcal{V} = \frac{1}{9\sqrt{2}} \left(\tau_b^{\frac{3}{2}} - \tau_s^{\frac{3}{2}} \right)\ . \tag{2.9}$$

Since the minimum of the scalar potential is expected to occur at large values of $\mathcal{V}$, the leading order instanton contribution $W_{\rm np} = A_s\, e^{-a_s T_s}$ is given by an E3-brane instanton along the small cycle. Furthermore, because $\mathcal{V} \gg 1$, one can perform an expansion of the potential V_F in powers of $1/\mathcal{V}$ [16]

$$\begin{aligned} V_F &= e^K \left(G^{a\bar{b}}\, D_a W\, D_{\bar{b}} \overline{W} - 3\big|W\big|^2 \right) \\ &= \lambda\, \frac{(a_s A_s)^2 \sqrt{\hat{\tau}_s}\, e^{-2a_s\hat{\tau}_s}}{\hat{\mathcal{V}}} - \mu\, \frac{a_s\big|A_s W_0\big|\, \hat{\tau}_s\, e^{-a_s\hat{\tau}_s}}{\hat{\mathcal{V}}^2} + \nu\, \frac{\xi\, \big|W_0\big|^2}{g_s^{3/2}\, \hat{\mathcal{V}}^3} + \ldots\ . \end{aligned} \tag{2.10}$$

Here λ, μ, ν are positive numerical constants and a_s takes the value 2π. The complex structure moduli U and the axio-dilation are assumed to be stabilised via $D_U W = D_S W =$

0 and sub-leading powers of $1/\hat{\mathcal{V}}$ have been neglected. For $\xi > 0$ this potential then has a minimum which stabilises $\hat{\mathcal{V}} \simeq \hat{\tau}_b^{3/2}$ at large values and $\hat{\tau}_s$ at $\hat{\tau}_s \simeq \log(\hat{\mathcal{V}})$. Introducing the mass scale $M_{\rm Pl}$, in this scenario one obtains the relation between the string and the Planck scale as $M_s \simeq M_{\rm Pl}/\sqrt{\mathcal{V}}$ and the relation $M_{\frac{3}{2}} \simeq M_{\rm Pl}\, W_0/\mathcal{V}$ for the gravitino mass.

In subsequent papers [17, 33, 18, 34, 19], many phenomenologically appealing features of this scenario have been found, which at the moment make them very attractive candidates for string phenomenology. Moreover, it has been shown in [35, 36] that due to a second no-scale structure string loop corrections to the Kähler potential are sub-dominant.

To prepare our following discussion let us mention two important points:

- In order for an E3-brane instanton to actually generate a superpotential term like in (2.6), the zero mode structure must be of a special nature. For the example $\mathbb{P}_{[1,1,1,6,9]}$[18] it was shown that the small divisor τ_s has an F-theory lift to a six-dimensional divisor in the Calabi-Yau fourfold with $\chi(D, \mathcal{O}) = 1$. By the zero mode criterion derived in [28], an instanton along this cycle thus contributes to the superpotential.

- In all the phenomenological analysis of the model above, the MSSM D7-branes were assumed to also wrap the small cycle τ_s. Therefore, the implicit philosophy was, that one first freezes all closed string moduli by fluxes and instantons, and then adds an MSSM like D7-brane sector and computes the soft terms depending on the non-vanishing auxiliary F-fields via the usual supergravity formulas [37].

This latter practice is surely justified in a purely phenomenological approach, but, given the undoubted success of this scenario, we would like to look more closely whether such a procedure is indeed justified from the structure of string theory. Of course, there are apparent stringy consistency conditions, which are not shown to be really satisfied in this concrete model, like for instance tadpole cancellation conditions and the vanishing of the Freed-Witten anomalies [38] appearing if both H_3-form flux and D-branes are present. If these are violated then the theory would be inconsistent right away as in general anomalies would not be cancelled.[4] What we are after, however, is more subtle and relates to the coexistence of a chiral D-brane sector and a moduli freezing instanton sector.

2.2 Orientifolds with intersecting D7-branes

Let us now collect the general rules for computing the massless spectrum and the tadpole cancellation conditions for Type IIB orientifolds with O7- and O3-planes. As before, we are compactifying the Type IIB string on a Calabi-Yau three-fold $\mathcal{X}$ but now we also specify an orientifold projection. It is of the form $\Omega\sigma(-1)^{F_L}$ where Ω is the world-sheet parity operator, σ is an involution and F_L denotes the left-moving fermion number. Then, we introduce stacks of D7$_a$-branes wrapping four-cycles D_a in the Calabi-Yau manifold and

[4] In a T-dual Type IIA model, where the Standard Model moduli can also be fixed by fluxes, it would not only be the interplay between intersecting D6-branes and E2-instantons but also the generalised Freed-Witten [39, 23] anomalies governing the coexistence between the chiral gauge sector and the moduli stabilisation sector.

Representation	Multiplicity
$(\overline{N}_a, N_b)$	I_{ab}
(N_a, N_b)	$I_{a'b}$
A_a	$\frac{1}{2}(I_{a'a} + 2I_{\mathrm{O7a}})$
S_a	$\frac{1}{2}(I_{a'a} - 2I_{\mathrm{O7a}})$

Table 1: Chiral spectrum for intersecting D7-branes.

carrying gauge bundles V_a. Here we consider only orientifold projections leaving all four-cycles invariant, i.e. $h^-_{1,1} = 0$ and $h^+_{1,1} = h_{1,1}$. This implies that the orientifold projection acts as

$$\Omega_3 \to -\Omega_3 \ , \qquad D_a \to D_a \ , \qquad V_a \to V_a^\vee , \tag{2.11}$$

where Ω_3 denotes the holomorphic three-form of the Calabi-Yau. The fixed point locus of the involution σ defines a divisor D_{O7} around which the orientifold plane is wrapped. Note that on the O7-plane there is no gauge bundle so that formally we choose $V_{\mathrm{O7}} = \mathcal{O}$.

The chiral massless spectrum arising from open strings stretched between two D7-branes wrapping two four-cycles D_a and D_b and carrying gauge bundles V_a and V_b is determined by [40, 41]

$$I_{ab} = \int_{D_a \cap D_b} \Big(c_1(V_a) - c_1(V_b) \Big) = \int_{\mathcal{X}} \Big(c_1(V_a) - c_1(V_b) \Big) \wedge [D_a] \wedge [D_b] \ . \tag{2.12}$$

Here, the two-forms $[D_{a,b}]$ denote the Poincaré duals to the four-cycles $D_{a,b}$ and $c_1(V_{a,b})$ denote the first Chern classes of $V_{a,b}$. The rules for computing the chiral spectrum are summarised in table 1 where a prime denotes the orientifold image.

Having a chiral spectrum implies that one has to worry about anomalies. However, satisfying the tadpole cancellation condition for the D7-branes ensures that the spectrum is free of non-abelian gauge anomalies. For the present case it reads

$$\sum_a N_a \, D_a = 4 \, D_{\mathrm{O7}} \ , \tag{2.13}$$

where the sum is over all D7$_a$-branes. Note that we have presented the tadpole constraint on the orientifold quotient. In the ambient Calabi-Yau it is multiplied by a factor of two. In addition, there is the D3-brane tadpole which, again on the quotient, takes the following form

$$N_{\mathrm{D3}} + N_{\mathrm{flux}} - \sum_a N_a \int_{D_a} \mathrm{ch}_2(V_a) = \frac{N_{\mathrm{O3}}}{4} + \sum_a \frac{N_a}{24} \int_{D_a} \mathrm{c}_2(T_{D_a}) + \frac{1}{12} \int_{D_{\mathrm{O7}}} \mathrm{c}_2(T_{\mathrm{O7}}) \ , \tag{2.14}$$

where T_D denotes the tangential bundle of the divisor D and c_2 stands for the second Chern class while ch_2 denotes the second Chern character. Note that for a smooth divisor D the integral of the second Chern class over D is just the Euler-characteristic $\chi(D)$.

In the F-theory lift of such a model to a Calabi-Yau fourfold $\mathcal{Y}$, the right hand side of equation (2.14) is equal to $\chi(\mathcal{Y})/24$ [42]. For the simple solution of (2.13) with four D7-branes with trivial line bundle placed right on top of the O7-plane we have

$$N_{\rm D3} + N_{\rm flux} = \frac{N_{O3}}{4} + \frac{\chi(D_{\rm O7})}{4} \; . \tag{2.15}$$

The gauge group in this case is SO(8). For this special solution, the Calabi-Yau four-fold is given by the $\mathbb{Z}_2$ orbifold $\mathcal{Y} = (\mathcal{X} \times \mathbb{T}^2)/\mathbb{Z}_2$ where the $\mathbb{Z}_2$ acts as the holomorphic involution σ on the CY three-fold $\mathcal{X}$ and on the torus $\mathbb{T}^2$ as $z \to -z$. If we blow-up the $\mathbb{Z}_2$ singularities by gluing in $\mathbb{P}^1$s and take the four fixed points on T^2 into account, the Euler-characteristic of $\mathcal{Y}$ is computed as

$$\begin{aligned}\chi(\mathcal{Y}) &= \frac{1}{2}\Big(\chi(\mathcal{X}\times\mathbb{T}^2) - 4\,\chi(D_{\rm O7}) - 4N_{\rm O3}\Big) + 4\,\chi(\mathbb{P}^1)\chi(D_{\rm O7}) + 4\,\chi(\mathbb{P}^1)\,N_{\rm O3} \\ &= 24\,\left(\frac{N_{O3}}{4} + \frac{\chi(D_{\rm O7})}{4}\right) \; .\end{aligned} \tag{2.16}$$

For other non-trivial and in particular chiral solutions to the tadpole cancellation conditions, the F-theory four-fold is not explicitly known.[5]

3. Instantons and chirality

In this section, we are going to investigate the aforementioned interplay between a chiral theory realised by intersecting and magnetised D7-branes and the E3-brane instantons. More specifically, we assume that some version of the MSSM can be described by a configuration of D7-branes wrapping four-cycles in the Calabi-Yau manifold.

3.1 The chiral D7-brane sector

The formula for the chiral spectrum between two D7-branes (2.12) implies that in order to obtain chirality, it is necessary that at least one of the D7-branes carries a non-trivial $\mathrm{U}(N)$ gauge bundle. For our purposes, it is not crucial to have a complete MSSM sector, but we will just take one of the main features of the Standard Model, namely its chirality, and assume the minimal chiral configuration. We consider K stacks of N_a D7-branes wrapping the cycle D_a with vector bundle V_a. However, in order to avoid stability issues of higher rank vector bundles and vector bundle moduli, from now on we just choose line bundles $\mathcal{L}_a$ on the D7-branes.

For such chiral intersecting D-brane models, it is known that generically they contain anomalous $\mathrm{U}(1)$ gauge symmetries. For D7-branes, these anomalies are cancelled by the four-dimensional axions

$$\rho_a = \int_{D_a} C_4 \tag{3.1}$$

[5] Note that it is somewhat misleading to do Type IIB model building with the fourfold base F_{11}, as here implicitly the four-cycles wrapped by the D7-branes have already been fixed and the only freedom is to turn on gauge bundles on them.

arising from the dimensional reduction of the R-R four-form along the four-cycle D_a. Indeed, the Chern-Simons action for a D7-brane on a four-cycle D_a contains terms of the form

$$S_{\rm CS} \sim \int_{\mathbb{R}^{1,3}\times D_a} C_4 \wedge F \wedge F \; , \tag{3.2}$$

which give rise to the following Green-Schwarz couplings. First, there is the mass-term for the gauge field obtained by choosing two legs of C_4 along D_a and F to be the curvature of the internal line bundle $\mathcal{L}$. Second, the $\rho - A^2$ vertex arises from choosing all four legs of C_4 along D_a. Such a gauging of the axionic shift symmetry leads to a Fayet-Iliopoulos term for a $\mathrm{U}(1)$, which in our case turns out to be

$$\xi_a = \frac{1}{\hat{\mathcal{V}}} \int_{\mathcal{X}} c_1\big(\mathcal{L}_a\big) \wedge \big[D_a\big] \wedge \hat{J} \; . \tag{3.3}$$

Therefore, a chiral D7-brane sector necessarily gives rise to a D-term potential V_D of the following form

$$V_D = \sum_{a=1}^{K} \frac{1}{\mathrm{Re}\,(f_a)} \left(\sum_i Q_i^{(a)} \big|\phi_i\big|^2 - \,\xi_a \right)^2 \; , \tag{3.4}$$

where $M_{\rm Pl} = 1$ and $Q_i^{(a)}$ are the $\mathrm{U}(1)_a$ charges of the canonically normalised matter fields ϕ_i. Furthermore, $\mathrm{Re}\,(f_a)$ denotes the real part of the gauge kinetic function for the corresponding D-brane. It is effectively the DBI action of a supersymmetric E3-brane instanton along the cycle D_a and reads

$$\mathrm{Re}\,(f_a) = e^{-\phi}\,\frac{1}{2} \int_{D_a} J \wedge J - e^{-\phi} \int_{D_a} \mathrm{ch}_2\big(B + \mathcal{L}_a\big) = \hat{\tau}_a - \mathrm{Re}\big(S\big)\, c_a \; . \tag{3.5}$$

Here, c_a denotes the integrated second Chern character of $B + \mathcal{L}_a$ on the respective D7-brane and $\hat{\tau}_a$ is the (Einstein-frame) volume of D_a.

Note that this D-term is generically only of order $\mathcal{V}^{-2}$ in the volume expansion (2.10) so that an additional (natural) D-term supersymmetry breaking destabilises the large volume minimum found at order $\mathcal{V}^{-3}$. Therefore, for preserving the large volume minimum we will require that the D-term vanishes, i.e. $V_D = 0$. The other option is to allow for significant fine tuning and use this D-term in a hidden sector for up-lifting the AdS minimum to a small and positive vacuum energy [43, 44, 39, 21].

3.2 E3-brane instantons

We are now going to investigate E3-brane instanton effects in more detail. In particular, for the instantons to generate a contribution to the superpotential, the zero mode structure has to be of a certain type.

Note that for orientifold models where the D7-branes do not lie right on top of the O7-planes, i.e. for generally intersecting D-branes with non-trivial gauge bundles, the F-theory lift is not known and the computation of $\chi(D, \mathcal{O})$ for the uplifted E3-brane divisor cannot be performed. It is therefore much more convenient to perform the zero mode analysis directly in the Type IIB orientifold model, where we can rely on recent work on space-time instanton effects in D-brane models [45 – 52]. We will also comment on the case that we freeze the Kähler moduli by gaugino condensates on a stack of N_c D7-branes wrapping a four-cycle D_G.

- For contributing to the superpotential, a single (isolated) instanton must wrap a four-cycle invariant under the orientifold projection and must carry an $O(1)$ gauge symmetry [53 – 55]. In the case of $h^-_{1,1}(\mathcal{X}) = 0$, this implies that the instanton carries a trivial gauge bundle.

- Next, we have to worry about deformation zero modes of the E3-instanton. These are clearly absent, if the E3-brane wrapping the four-dimensional divisor D does not have any further moduli. That is, there are no Wilson lines counted by $H^1(D, \mathcal{O})$ or transverse deformations counted by $H^2(D, \mathcal{O})$. If this sufficient condition is not satisfied, then fluxes or curvature on the moduli space might soak up some of the zero modes, but a more careful analysis is necessary [51, 56]. Similarly, for gaugino condensation many adjoint matter fields counted by $H^i(D_G, \mathcal{O})$ with $i = 1, 2$ spoil asymptotic freedom of the gauge theory on the $D7_G$-branes.

- If, as in our case, there are additional space-time filling D7-branes present, there can appear extra charged fermionic zero modes from the intersection of the E3-instanton and the D7-branes [46]. The chiral index of these fermionic zero modes is

$$Z_a = N_a \int_{D_a \cap D_{\mathrm{E3}}} c_1(\mathcal{L}_a) = N_a \int_{\mathcal{X}} c_1(\mathcal{L}_a) \wedge [D_a] \wedge [D_{\mathrm{E3}}] \, . \tag{3.6}$$

  In order to soak up these additional fermionic zero modes, one has to pull down charged matter fields in the instanton computation. The pure exponential term as in (2.6) is then multiplied by products of charged matter superfields Φ_i as [46]

$$W_{\mathrm{string}} \sim \left[\prod_i \Phi_i \right] e^{-\mathcal{S}_{\mathrm{inst}}}. \tag{3.7}$$

  Note, that such instantons are not gauge instantons and therefore often called stringy or exotic instantons.

- For the special case when the E3-instanton lies right on top of the D7-branes,[6] it is possible to have non-trivial gauge bundles on the instanton. It can then be regarded as a gauge instanton from the perspective of the D7-brane gauge theory and additional bosonic and non-chiral fermionic zero modes arise parametrising the ADHM instanton super moduli space [45, 59, 53, 54]. The effect of such instantons is of the same nature as gaugino condensates for the gauge theory on stacks of $\mathrm{D7}_G$-branes, so that we can discuss them together. In order to soak up the ADHM zero modes one needs extra

[6]Note that in [57, 58] it was shown that for an instanton on top of a single D-brane also a superpotential of the form (3.7) can be generated.

non-chiral (with respect to the $\mathrm{U}(N_G)$ gauge group) matter zero modes from the intersection of the E3-instanton with the other D7-branes [35, 59]. If we end up with an $\mathrm{SU}(N_c)$ gauge group with effectively N_f flavours, then for $N_f < N_c$ the contribution to the superpotential is

$$W_{\text{gauge}} \sim \frac{1}{\det_{ff'}\left[\widetilde{\Phi}_f{}^c\, \Phi_{c\,f'}\right]}\, e^{-\mathcal{S}_{\text{inst}}} \,. \tag{3.8}$$

In writing this, it is assumed that we are on the Higgs branch, where the determinant is non-vanishing and so the flavour gauge group is completely broken. Such a configuration is not part of the MSSM and therefore the instanton respectively the D_G branes should better not have any intersection with the D-branes supporting the MSSM.

3.3 Moduli stabilisation for chiral models

We will now argue that given the structure and constraints from the previous discussion, for chiral orientifolds not all Kähler moduli can be frozen by instantons. In particular, some of the moduli controlling the size of the chiral D7-brane sector are left unfixed by the E3-brane instantons.

Let us first summarise the possible matter fields which can be present in the configurations we are considering.

- We assume that the chiral MSSM like matter fields, denoted as Φ_{SM}, are part of the chiral matter spectrum arising on a set of intersecting D7-branes carrying initial gauge group $G = \prod_{a=1}^{K} \mathrm{U}(N_a)$. Typical examples discussed in the literature are $G = \mathrm{U}(5) \times \mathrm{U}(1)$, $G = \mathrm{U}(4) \times \mathrm{U}(2) \times \mathrm{U}(2)$ or $G = \mathrm{U}(3) \times \mathrm{U}(2) \times \mathrm{U}(1) \times \mathrm{U}(1)$.
- There can also be additional (chiral) fields, which also arise from the same set of intersecting D7-branes leading to so-called exotic matter fields. There can exist exotic matter fields transforming in non-trivial representation of the non-abelian part of the MSSM gauge group. These are denoted as Φ_{exo}.
- However, since in D-brane models we genuinely have these extra $\mathrm{U}(1)$ gauge factors, there might be fields which are not charged under the MSSM gauge group $\mathrm{SU}(3) \times \mathrm{SU}(2) \times \mathrm{U}(1)_Y$ but carry non-trivial charges with respect to $\mathrm{U}(1)$s orthogonal to $\mathrm{U}(1)_Y$. These we denote as Φ_{abel}.
- In addition there can in principle be further hidden sector matter fields Φ_{H}, whose D-terms and F-terms however do not mix with the Standard Model ones. Therefore, we will not focus on those in the following. However, this sector might be important for the eventual uplift of the AdS minimum to de Sitter with small cosmological constant.

Consider now an E3-instanton wrapping a four-cycle which gives rise to extra Standard Model charged zero modes. These can either be chiral fermionic zero modes coming from stringy instantons or non-chiral zero modes from gauge instantons. To soak up all these zero modes, the superpotential coupling must contain products of the Standard Model Matter fields $\Phi_{\rm SM}$ and, since they appear on the same D7-branes, also products of the additional fields $\Phi_{\rm exo}$ and $\Phi_{\rm abel}$

$$W \sim \prod_i \Phi_{\rm SM}^{(i)} \prod_j \Phi_{\rm exo}^{(j)} \prod_k \Phi_{\rm abel}^{(k)} \; e^{-T_{E3}} \, . \tag{3.9}$$

Note that for gauge instantons or gaugino condensates there will be determinants of the matter fields in the denominator. Furthermore, in the equation above $T_{\rm E3} = \sum_i m^i T_i$ denotes the Kähler modulus corresponding to the instanton on the cycle $D_{\rm E3} = \sum_i m^i D_i$.

The important point is now that, for phenomenological reasons, at this high scale we do not want to break the MSSM gauge symmetry by giving VEVs to these fields. If we allow for VEVs of charged matter fields, the D-term potential (3.4) generates a mass of the generic order $M_{\rm matter} = M_{\rm Pl}/\sqrt{\mathcal{V}} = M_s$ for them, i.e. the matter fields become very heavy. The MSSM gauge symmetry breaking and mass generation should occur as usual at the low scale in the process of supersymmetry breaking. Therefore, we are only interested in vacua with $\langle \Phi_{\rm SM} \rangle = \langle \Phi_{\rm exo} \rangle = 0$, so that effectively the contribution of such an instanton to the superpotential vanishes and the F-term potential V_F does not depend explicitly on $T_{\rm E3}$. What could be possible in principle is to allow VEVs for GUT Higgs fields.

Of course this argumentation is not really satisfying as in a fully realistic moduli stabilisation scenario, we also would like to have these charged matter fields dynamically stabilised. But our point of view is, that it is very likely that in a given concrete model the four contributions:[7]

- the soft supersymmetry breaking mass terms $V_{\rm soft} = m^2 \Phi_{\rm SM}^2$,
- the perturbative and instanton induced superpotential contributions of the form $W = \prod \Phi_{\rm SM}$,
- the D-terms and
- the generic absence of gauge instantons or gaugino condensates for MSSM fields, i.e. terms like $W_{\rm gauge} \sim \frac{1}{\det[\Phi_{\rm SM}]} \, e^{-\mathcal{S}_{\rm inst}}$

suffice to freeze to MSSM matter fields at $\langle \Phi_{\rm SM} \rangle = \langle \Phi_{\rm exo} \rangle = 0$. If such a mechanism is indeed at work, then, since they appear in the same open string sector, also the fields $\Phi_{\rm abel}$ are likely to be frozen at vanishing VEVs. However, just from phenomenology these VEVs could be non-vanishing, a fact to be kept in mind when we will mainly discuss the case $\langle \Phi_{\rm abel} \rangle = 0$.

Therefore, if we want to fix the size of the four-cycle the E3-instanton is wrapping, it should not have any zero modes charged under the Standard Model gauge symmetry. Recall that we derived the analogous condition also for moduli freezing via gaugino condensates on a stack of D7-branes wrapping a four-cycle D_G. There too, D_G should not have any charged matter fields from intersections with branes supporting the MSSM.

[7]See for instance [60, 61] for a recent discussion of matter fields moduli stabilisation.

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Recalling then equation (3.6), we have to satisfy the necessary condition

$$N_a \int_{\mathcal{X}} c_1(\mathcal{L}_a) \wedge [D_a] \wedge [D_{\mathrm{E3}}] = 0 \,, \tag{3.10}$$

for Standard Model branes wrapping the divisor D_a with line bundle $\mathcal{L}_a$. Furthermore, not only the chiral instanton zero modes have to be absent but also those which are vector-like. For determining them one has to compute the cohomology classes

$$H^i \left(D_a \cap D_{\mathrm{E3}} \,,\, \mathcal{L}_a \otimes \mathcal{K}_{D_a}^{\frac{1}{2}} \otimes \mathcal{K}_{D_{\mathrm{E3}}}^{\frac{1}{2}} \right) \qquad \text{for} \quad i = 0, 1 \,, \tag{3.11}$$

where $\mathcal{K}_D$ denotes the canonical line bundle of the divisor $D \subset \mathcal{X}$. If these cohomology classes are non-trivial, extra pairs of instanton zero modes are present and the resulting term in the superpotential will be of the form (3.9). However, in this paper we will mainly be concerned with chiral zero modes and generically do not explicitly determine the vector-like ones. But one has to keep in mind that they might be present and one has to worry about soaking them up.

Coming back to equation (3.10), we can expand the Poincaré dual of the instanton cycle $[D_{\mathrm{E3}}]$ in a basis $\{\omega_i\}$ of two-forms in $H^{1,1}(\mathcal{X})$

$$[D_{\mathrm{E3}}] = \sum_i m^i \, \omega_i \,. \tag{3.12}$$

Then, we define the following matrix

$$\mathcal{M}_{a,i} = \int_{\mathcal{X}} c_1(\mathcal{L}_a) \wedge [D_a] \wedge \omega_i \,, \tag{3.13}$$

with $i = 1, \ldots, h_{1,1}(\mathcal{X})$ and $a = 1, \ldots, K$ where K is the number of MSSM supporting D7-branes carrying $\mathrm{U}(N)$ gauge symmetry . To not over-constrain the system, we can assume that $K \leq h_{1,1}$ and so the maximal number of linear independent E3-brane instantons $\mathcal{N}_{\mathrm{E3}}$ one is allowed to introduce is given by the kernel of the matrix $\mathcal{M}_{a,i}$.

Since the kernel of the matrix (3.13) is not equal to $h_{1,1}(\mathcal{X})$ because of the chirality of the MSSM, it is clear that not all Kähler moduli can be stabilised by E3-brane instantons. But let us expand the Kähler form J in the basis $\{\omega_i\}$ as $J = \sum_i t^i \, \omega_i$. Recalling then equation (3.3), we find that the Fayet-Iliopoulos parameter can be expressed as

$$\xi_a = \frac{1}{\hat{\mathcal{V}}} \int_{\mathcal{X}} c_1(\mathcal{L}_a) \wedge [D_a] \wedge \hat{J} = \frac{1}{\hat{\mathcal{V}}} \sum_i \mathcal{M}_{a,i} \, \hat{t}^i \,, \tag{3.14}$$

so that the Kähler moduli will also appear in the D-terms. The vanishing of the D-terms then provides additional restrictions on the t^i. The number of moduli fixed through these equations is given by the rank of the matrix $\mathcal{M}_{a,i}$ which satisfies $\mathrm{rk}(\mathcal{M}) \geq K_{\mathrm{anom}}$ where K_{anom} denotes the number of anomalous $\mathrm{U}(1)$ gauge factors supported on the MSSM branes. To be more precise, the Kähler moduli counted by the defect of $\mathcal{M}_{a,i}$ are fixed by the D-term. These are orthogonal to the ones possibly fixed by E3-instantons and are in the kernel of $\mathcal{M}_{a,i}$. Since the MSSM matter spectrum is chiral, it is clear from the definition of $\mathcal{M}_{a,i}$ that there must be at least one anomalous $\mathrm{U}(1)$ gauge factor.

To summarise: at most $h_{1,1} - K_{\rm anom}$ Kähler moduli can be fixed by E3-instantons whereas for the remaining moduli, which control the size of the D7-branes supporting the MSSM sector, there appears a D-term potential. For not destabilising the large volume minimum due to the $1/\mathcal{V}^2$ factor in front, this D-term has to vanish. Therefore, despite our initial concern, with sufficient rigid instantons being present in a model, we have enough constraints to fix all Kähler moduli. If we cannot fix all remaining Kähler moduli via instantons and D-terms, there also exist the possibility that they are frozen similar to $\mathcal{V}$ by perturbative corrections to the F-term scalar potential. Arguments have been given that this should occur for the QCD axion [62].[8]

Clearly, the general arguments presented above need to be investigated more carefully for each model, this is however beyond the scope of this paper. From now on, if not dynamically proven but at least phenomenologically motivated, we generally assume

$$\langle \Phi_{\rm SM} \rangle = \langle \Phi_{\rm exo} \rangle = \langle \Phi_{\rm abel} \rangle = 0 \,, \tag{3.15}$$

so that the vanishing of the D-terms in the MSSM sector effectively implies the vanishing of the Fayet-Iliopoulos parameters (3.14). We will mention at certain points the changes once VEVs of $\Phi_{\rm abel}$ are nonvanishingg, but as we stressed already so far we do not have a complete theory to dynamically freeze these moduli.

3.4 F-term scalar potential

In the original work about the large volume scenario [16], only the case with one E3-brane instanton along one small four-cycle was studied in detail. Later it was argued that similar results carry over to configurations where more than one four-cycle stays small supporting instantons [29]. For our purpose it is useful and illustrative to start again from a general setup and perform the steps along the lines of [16].

Similarly to section 2.1, we assume that the complex structure moduli U and the axio-dilaton S have been fixed by fluxes via $D_U W = D_S W = 0$ and the value of the Gukov-Vafa-Witten superpotential (2.2) in the minimum will again be denoted by W_0. For the stabilisation of the Kähler moduli we use the usual α'-corrected Kähler potential (2.3) and introduce E3-instantons. However, we we allow for instantons wrapping general four-cycles $D_\alpha = M^i_\alpha D_i$ where M^i_α are the wrapping numbers of the instanton α and $\{D_i\}$ is a basis of four-cycles on $\mathcal{X}$. The superpotential then takes the form

$$W = W_0 + \sum_\alpha A_\alpha \, e^{-2\pi M^i_\alpha T_i} \,, \tag{3.16}$$

where the sum is over all contributing instantons in the large radius limit. Computing the

[8] We thank Joe Conlon for bringing this to our attention.

Kähler metric similarly to [63], we can write the scalar F-term potential as

$$\begin{aligned} V_F = e^K \Bigg(& -\frac{(2\pi)^2}{2} \left(2\hat{\mathcal{V}} + \hat{\xi}\right) \sum_{\alpha,\beta} \mathrm{Vol}\big(D_\alpha \cap D_\beta\big)\, A_\alpha\, \overline{A}_\beta\, e^{-2\pi M^i_\alpha T_i}\, e^{-2\pi M^j_\beta \overline{T}_j} \\ & + \frac{(2\pi)^2}{4}\, \frac{4\hat{\mathcal{V}} - \hat{\xi}}{\hat{\mathcal{V}} - \hat{\xi}} \sum_{\alpha,\beta} \hat{\tau}_\alpha\, \hat{\tau}_\beta\, A_\alpha\, \overline{A}_\beta\, e^{-2\pi M^i_\alpha T_i}\, e^{-2\pi M^j_\beta \overline{T}_j} \\ & + \frac{2\pi}{2}\, \frac{4\hat{\mathcal{V}}^2 + \hat{\mathcal{V}}\hat{\xi} + 4\hat{\xi}^2}{\big(2\hat{\mathcal{V}} + \hat{\xi}\big)\big(\hat{\mathcal{V}} - \hat{\xi}\big)} \sum_\alpha \hat{\tau}_\alpha \left(A_\alpha\, e^{-2\pi M^i_\alpha T_i} \overline{W} + \overline{A}_\alpha\, e^{-2\pi M^i_\alpha \overline{T}_i} W\right) \\ & + 3\,\hat{\xi}\, \frac{\hat{\mathcal{V}}^2 + 7\hat{\mathcal{V}}\hat{\xi} + \hat{\xi}^2}{\big(2\hat{\mathcal{V}} + \hat{\xi}\big)^2 \big(\hat{\mathcal{V}} - \hat{\xi}\big)}\, \big|W\big|^2 \quad \Bigg)\,. \end{aligned} \tag{3.17}$$

Here we have used $\hat{\mathcal{V}}$ and $\hat{\tau}_\alpha$ to respectively denote in Einstein-frame the volume of the Calabi-Yau manifold and the volume of the four-cycle wrapped by the instanton α. Furthermore, to simplify the formulas we used

$$\mathrm{Vol}\big(D_\alpha \cap D_\beta\big) = M^i_\alpha\, M^j_\beta\, \mathcal{K}_{ijk} \hat{t}^k \tag{3.18}$$

for the volume of the intersection of two four-cycles D_α and D_β (in Einstein-frame) and we have defined $\hat{\xi} = \xi / g_s^{3/2}$.

Let us now perform the large volume expansion of V_F. Note that in this limit the second term in (3.17) is sub-leading. Keeping also only the leading term W_0 in the superpotential, we find up to an overall constant

$$\begin{aligned} V_F \simeq & -\frac{(2\pi)^2}{\hat{\mathcal{V}}} \sum_{\alpha,\beta} \mathrm{Vol}\big(D_\alpha \cap D_\beta\big)\, A_\alpha\, \overline{A}_\beta\, e^{-2\pi M^i_\alpha T_i}\, e^{-2\pi M^j_\beta \overline{T}_j} \\ & + \frac{2\pi}{\hat{\mathcal{V}}^2} \sum_\alpha \hat{\tau}_\alpha \left(A_\alpha\, e^{-2\pi M^i_\alpha T_i}\, \overline{W}_0 + \overline{A}_\alpha\, e^{-2\pi M^i_\alpha \overline{T}_i}\, W_0\right) + \frac{3}{4}\, \frac{\hat{\xi}}{\hat{\mathcal{V}}^3}\, \big|W_0\big|^2\,. \end{aligned} \tag{3.19}$$

In the one instanton case, the second term in equation (3.19) was the only place where the axion corresponding to the instanton appeared. Recalling $T_i = \hat{\tau}_i + i\rho_i$, such a term could be written as $X e^{i\rho} + \overline{X} e^{-i\rho}$ and upon minimising the potential with respect to ρ, it was rendered real and negative [16, 17]. The negativity of this term was crucial for the existence of the minimum of the F-term potential at exponentially large volume.

In the general case of more than one instanton, the first term in (3.19) also depends on the axions, provided the volume of the intersection locus of the respective instanton-cycles is non-vanishing. In this case, a more careful analysis of V_F is needed, which we leave for future work [64]. Requiring though that

$$\mathrm{Vol}\big(D_\alpha \cap D_\beta\big) = 0 \tag{3.20}$$

for all pairs of instantons with $\alpha \neq \beta$, guarantees that the respective axions are stabilised in the way described above by the second term in (3.19).

For the following, we will restrict ourselves to the case of one instanton wrapping a general four-cycle D_{E3} in the Calabi-Yau manifold. Employing then the stabilisation of the axion associated to the instanton illustrated above, the F-term potential for one E3-brane instanton simplifies to

$$V_F \simeq -\frac{(2\pi)^2}{\hat{\mathcal{V}}}\,\mathrm{Vol}\big(D_{\mathrm{E3}}\cap D_{\mathrm{E3}}\big)\,\big|A_{\mathrm{E3}}\big|^2\, e^{-4\pi\hat{\tau}_{\mathrm{E3}}} - \frac{4\pi}{\hat{\mathcal{V}}^2}\,\hat{\tau}_{\mathrm{E3}}\, e^{-2\pi\hat{\tau}_{\mathrm{E3}}}\,\big|A_{\mathrm{E3}}W_0\big| + \frac{3}{4}\,\frac{\hat{\xi}}{\hat{\mathcal{V}}^3}\,\big|W_0\big|^2. \tag{3.21}$$

This expression is nearly similar to the well-known expression of V_F (2.10) in the original large volume scenario. The only difference is the first term. If we find that

$$\mathrm{Vol}\big(D_{\mathrm{E3}}\cap D_{\mathrm{E3}}\big) \simeq -\sqrt{\hat{\tau}_{\mathrm{E3}}}\,, \tag{3.22}$$

then as shown in [16], we are guaranteed to find a minimum of V_F at exponentially large values of $\mathcal{V}$ and with $\tau_{\mathrm{E3}} \simeq \log(\mathcal{V})$. However, in general the minima of V_F will depend on the concrete model and on the way the moduli are stabilized.

Let us summarize the results of this part. Performing the large volume expansion of the scalar F-term potential for a general instanton configuration leads to an expression where the axions corresponding to the instantons cannot be stabilized easily. We did not attempt to address this question but restricted us to the case of one instanton along a general four-cycle.

The main question is now whether it is indeed possible to freeze the Kähler moduli controlling the size of the MSSM D7-branes via the D-terms of the U(1) gauge factors supported on these D7-branes and whether these sizes are of the same order of magnitude as the instantonic four-cycles. Let us collect the formal constraints we have to successfully implement in a concrete model for this scenario to work:

- Find a Calabi-Yau of swiss-cheese type with one large four-cycle controlling the size of the manifold and small cycles typically arising from resolutions of singularities.[9]
- Define an orientifold projection of this space leading to O7- and O3-planes and freeze the complex structure and dilaton moduli by G_3-form flux. This latter will contribute to the D3-brane tadpole.
- Introduce a set of intersecting (magnetised) D7-branes supporting the chiral MSSM spectrum and a hidden D7-brane sector such that the D7- and D3-brane tadpole cancellation conditions are satisfied. Moreover, the D7-branes must be free of Freed-Witten anomalies [38].
- Classify all E3-instantons on this space which from the zero mode structure can contribute to the uncharged superpotential. For this, a sufficient condition is that the instanton is rigid and has no other chiral or vector-like zero modes from E3-D7 intersections. Furthermore, one also needs to ensure that the instantons are free of Freed-Witten anomalies [65].

[9]It would be interesting to investigate whether also for instance Calabi-Yaus with a fibration structure can lead to large volume moduli freezing. For these the volume can usually be brought to the schematic form $\mathcal{V} = \tau_1\sqrt{\tau_2} - \sum_I \tau_I^{\frac{3}{2}}$.

- Compute the effective F- and D-term potential and analyse whether the combination of both freezes all Kähler moduli inside the Kähler cone with the size of the D7-branes coming out of the same order as the sizes of the instantons $\tau \simeq \log(\mathcal{V})$.

Moreover, since in the non-supersymmetric large volume minimum the D-terms vanish, we still only have F-term supersymmetry breaking and the soft-terms can be computed in the usual way [37].

In the remainder of this paper, we will explicitly carry out some of the steps mentioned above for a concrete Calabi-Yau orientifold model. Our simple (toy) model is neither realistic nor can all conditions mentioned above be met explicitly, but it nevertheless shows how this program can partly be realised even on a simple Calabi-Yau manifold. We leave a more phenomenological discussion of this moduli freezing scenario for future work.

4. The $\mathbb{P}_{[1,3,3,3,5]}[15]$ Calabi-Yau

For the large volume scenarios reviewed in section 2.1 it is now clear that we need at least three Kähler moduli to have both E3-instantons and a chiral D7-brane sector. The exponentially large cycle, controlling the overall size of the manifold, is usually frozen by the competing effects of the leading order α'-corrections to the Kähler potential and the E3-instanton contribution. On the small cycles of a swiss-cheese type Calabi-Yau, the instantons and the D7-branes will be distributed.

Checking some Calabi-Yau three-folds defined as hypersurfaces in weighted projective spaces, we found one candidate which actually is of swiss-cheese type. It is the resolution of the $\mathbb{P}_{[1,3,3,3,5]}[15]$ manifold. It will turn out that this Calabi-Yau is still not rich enough to allow for complex structure moduli stabilisation by fluxes and a complete MSSM sector, but serves as a simply toy model to give a proof of principle how the combination of F- and D-term moduli stabilisation can work in more realistic models. Let us describe the algebraic geometry of this Calabi-Yau in some more detail in the next subsections.

4.1 The topology of $\mathbb{P}_{[1,3,3,3,5]}[15]$

Toric resolution. The $\mathbb{P}_{[1,3,3,3,5]}[15]$ manifold has a $\mathbb{Z}_3$ singularity along the complex line $x_1 = x_5 = 0$, which is met by the hypersurface constraint. The resolution of this A_2 orbifold singularity introduces two intersecting $\mathbb{P}^1$s over the line.

This resolution is easily described invoking the methods of toric geometry. Besides the five divisors $v_1^* = (1,0,0,0)$, $v_2^* = (0,1,0,0)$, $v_3^* = (0,0,1,0)$, $v_4^* = (0,0,0,1)$, $v_5^* = (-3,-3,-3,-5)$ one introduces the two blowing-up divisors $v_6^* = (-2,-2,-2,3)$ and $v_7^* = (-1,-1,-1,-1)$. The unique maximal triangulation is then given by

$$\begin{aligned}\text{Triangle} = \big\{&[1,2,3,4],[1,2,3,5],[1,2,4,7],[1,2,6,7],[1,2,5,6],[1,3,4,7],\\ &[1,3,6,7],[1,3,5,6],[2,3,4,7],[2,3,6,7],[2,3,5,6]\big\}\,.\end{aligned} \tag{4.1}$$

The data of the associated linear sigma model is the following. We have seven complex coordinates x_i with three $\mathrm{U}(1)$ symmetries. The corresponding charges are shown in (4.2).

x_1	x_2	x_3	x_4	x_5	x_6	x_7	p
3	3	3	5	1	0	0	15
2	2	2	3	0	1	0	10
1	1	1	1	0	0	1	5

(4.2)

The divisors D_i are defined by the constraints $x_i = 0$ and the resulting Stanley-Reisner ideal reads[10]

$$SR = \left\{ x_4\, x_5 \,,\; x_4\, x_6 \,,\; x_5\, x_7 \,,\; x_1\, x_2\, x_3\, x_6 \,,\; x_1\, x_2\, x_3\, x_7 \right\} . \tag{4.3}$$

The triple intersection numbers in the basis $\eta_1 = D_5$, $\eta_2 = D_6$, $\eta_3 = D_7$ are calculated as

$$I_3 = 9\,\eta_1^3 - 40\,\eta_2^3 - 40\,\eta_3^3 - 15\,\eta_1^2\eta_2 + 25\,\eta_1\eta_2^2 - 5\,\eta_2^2\eta_3 + 15\,\eta_2\eta_3^2\,. \tag{4.4}$$

From section 2.1 we recall that the volume τ_i of the divisor D_i and the overall volume of the manifold (in string-frame) are expressed in terms of the Kähler form in the following way

$$\tau_i = \frac{1}{2}\int_{\mathcal{X}} [D_i] \wedge J \wedge J\,, \qquad \mathcal{V} = \frac{1}{6}\int_{\mathcal{X}} J \wedge J \wedge J\,. \tag{4.5}$$

Expanding then the Kähler form in the basis $\{\eta_1, \eta_2, \eta_3\}$ from above as $J = \sum_{i=1}^{3} t_i\,[\eta_i]$ we find for the volumes of the divisors D_5, D_6 and D_7

$$\begin{aligned}
\tau_5 &= \frac{1}{2}\Big(3\,t_1 - 5\,t_2\Big)^2\,, \\
\tau_6 &= \frac{5}{6}\left[\Big(3\,t_3 - t_2\Big)^2 - \Big(5\,t_2 - 3\,t_1\Big)^2\right]\,, \\
\tau_7 &= -\frac{5}{2}\Big(t_2 - 4\,t_3\Big)\Big(t_2 - 2\,t_3\Big)\,.
\end{aligned} \tag{4.6}$$

The Kähler cone. Next, we are going to determine the Kähler cone, which is defined by the condition that the volumes of all effective curves $\mathcal{C}$ are positive. The first step is to compute the cone of all effective curves, which is called the Mori cone and then deduce from this the Kähler cone by the condition $\int_{\mathcal{C}} J > 0$. The resulting constraints describing the Kähler cone are

$$t_2 - 2\,t_3 > 0\,, \qquad t_1 - 2\,t_2 + t_3 > 0\,, \qquad -3\,t_1 + 5\,t_2 > 0\,. \tag{4.7}$$

These conditions ensure also that the overall volume $\mathcal{V}$ is positive, that all volumes of effective divisors are positive and, by construction, that all volumes of holomorphic curves are positive.

[10]We used the maple package "Schubert" to perform part of these computations.

Swiss-cheese structure. For a large volume compactification we want to make one four-cycle large while keeping the others small. Let us therefore take a closer look at the volume. Using the Kähler cone restrictions above, we find that $\mathcal{V}$ can be written as

$$\mathcal{V} = \sqrt{\frac{2}{45}} \left(\left(5\tau_5 + 3\tau_6 + \tau_7\right)^{3/2} - \tfrac{1}{3} \left(5\tau_5 + 3\tau_6\right)^{3/2} - \tfrac{\sqrt{5}}{3} \left(\tau_5\right)^{3/2} \right) . \tag{4.8}$$

From this expression we see that this model admits a swiss-cheese structure. Indeed, we can make τ_7 large so that the total volume $\mathcal{V}$ becomes large while keeping the four-cycles volumes τ_5 and τ_6 small. On the latter ones the D-branes supporting the MSSM will be wrapped.

In such a setup, we are thus not allowed to wrap D-branes supporting the MSSM on (some combination involving) D_7 because then the gauge coupling $1/g^2_{\rm YM} \sim \tau_7$ would be too small. Similarly, we ignore instantons along this divisor, because its contribution to the superpotential is exponentially suppressed. We are then left with the two divisors D_5 and D_6. Note however that not all combinations of D_5 and D_6 are allowed. We have to wrap D-branes and instantons along effective cycles, i.e. positive linear combinations of the divisors.

Rigid cycles. Furthermore, we require the instanton to be rigid in the sense that no extra fermionic zero modes from the deformations of the cycle or from Wilson lines along one-cycles do appear. The transverse deformations of a holomorphic four-cycle D are counted by the global sections of the normal bundle N of D. By the adjunction formula and Serre duality on D we get $H^0(D, N_D) = H^2(D, \mathcal{O}_D)$. The Wilson lines are counted by the non-contractable one-cycles on D, which are counted by $H^1(D, \mathcal{O}_D)$. Therefore, for an instanton to not have additional deformation zero modes we will require

$$H^0\big(D, \mathcal{O}_D\big) = 1 \, , \qquad H^i\big(D, \mathcal{O}_D\big) = 0 \, , \qquad \text{for} \quad i = 1, 2 \, . \tag{4.9}$$

A necessary criterion for this is that the Euler characteristic of the trivial line bundle over D is equal to one, i.e.

$$\chi\big(D, \mathcal{O}_D\big) = \sum_{i=0}^{2} (-1)^i \, H^i(D, \mathcal{O}_D) = 1 \, . \tag{4.10}$$

Employing the Koszul sequence

$$0 \to \mathcal{O}_{\mathcal{X}}[-D] \to \mathcal{O}_{\mathcal{X}} \to \mathcal{O}_D \to 0 \, , \tag{4.11}$$

and the resulting long exact sequence in cohomology, one obtains the relation $\chi(D, \mathcal{O}_D) = \chi(\mathcal{X}, \mathcal{O}[-D])$.

In our concrete example, for a four cycle $D = m\,\eta_1 + n\,\eta_2 + l\,\eta_3$ the Euler characteristic is calculated as

$$\begin{aligned} \chi\big(D, \mathcal{O}_D\big) \; &= \; \frac{15}{2} n l^2 + \frac{25}{2} m n^2 - \frac{5}{2} n^2 l - \frac{15}{2} m^2 n - \frac{20}{3} l^3 \\ &\quad - \frac{20}{3} n^3 + \frac{5}{3} n + \frac{5}{3} l + \frac{3}{2} m^3 - \frac{1}{2} m \, . \end{aligned} \tag{4.12}$$

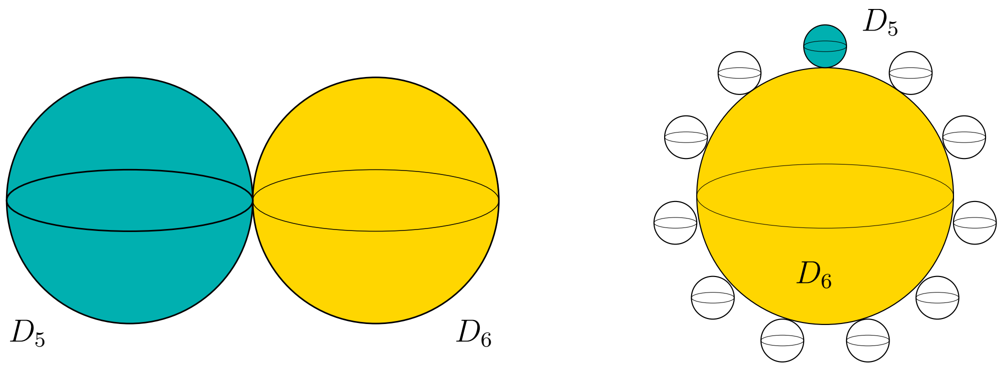

Figure 1: Singular rigid divisors.

Looking via a computer search for combinations with $\chi(D, \mathcal{O}_D) = 1$ and $l = 0$ we have found the solutions

$$(m, n, l) = \big\{(1,0,0)\ ,\ \ (1,1,0)\ ,\ \ (2,1,0)\ ,\ \ (2,2,0)\ ,\ \ (12,11,0)\big\}\ . \tag{4.13}$$

In order to compute the precise cohomology classes $H^i(D, \mathcal{O}_D)$, we use the cohomology classes of general line bundles on the toric ambient space shown in appendix A and then run them through the Koszul sequences for the restrictions on the Calabi-Yau hypersurface and the divisors D. The result is that the first four divisors in (4.13) really have $H^i(D, \mathcal{O}_D) = (1,0,0)$, i.e. these are irreducible effective divisors without any Wilson lines or transverse deformations.

One comment is in order here. Note that the three rigid divisors $(1,1,0)$, $(2,1,0)$, $(2,2,0)$ are singular. Let us explain this for the first one $D_5 + D_6$. The only constraint one can write down of this degree is $Q = x_5\, x_6 = 0$. This defines two complex divisors $x_5 = 0$ and $x_6 = 0$ intersecting along the curve $x_5 = x_6 = 0$, where the manifold becomes singular. Since the four-cycle has no deformations, the singularity cannot be smoothed out. A lower dimensional analogy is shown in figure 1. In the following we allow E3-instantons and D7-branes to also wrap these rigid cycles.[11] Once we will have specified our orientifold projection, we will show that all these rigid cycles carry SP gauge group for D7-branes wrapped around them and consequently SO gauge group for wrapped E3-instantons.

A similar structure appears for the rigid cycle $2D_5 + D_6$. By computing $H^i(2D_5, \mathcal{O})) = (11,0,0)$, we find that the instanton divisor has the structure shown figure 1. The intersection between D_6 and any of the eleven components of $2D_5$ is over a two cycle.

Diagonal basis. In the following it will be more convenient to work in a basis where the volume $\mathcal{V}$ as well as the triple intersection numbers become particularly simple. Guided by (4.8), we introduce the new basis of divisors as

$$D_a = 5\,D_5 + 3\,D_6 + D_7\ , \qquad D_b = 5\,D_5 + 3\,D_6\ , \qquad D_c = D_5\ , \tag{4.14}$$

[11] We have been informed by Volker Braun, that they have identified such topologies of world-sheet instantons to contribute to the heterotic superpotential [66 – 68].

for which the triple intersection numbers diagonalise

$$I_3 = 5\,D_a^3 + 45\,D_b^3 + 9\,D_c^3\,. \tag{4.15}$$

The total volume in terms of the divisor volumes τ_a, τ_b and τ_c reads

$$\mathcal{V} = \sqrt{\frac{2}{45}}\left(\tau_a^{3/2} - \frac{1}{3}\tau_b^{3/2} - \frac{\sqrt{5}}{3}\tau_c^{3/2}\right)\,. \tag{4.16}$$

Expanding also the Kähler form in this diagonal basis as $J = t_a\,D_a - t_b\,D_b - t_c\,D_c$, we find that the Kähler cone conditions have the very simple form

$$\frac{1}{3}\,t_a > t_b > t_c > 0\,. \tag{4.17}$$

As one can see from the above, the large divisor is now simply D_a. For the gauge couplings not to be unrealistically small, we do not wrap the D7-branes supporting the MSSM along the large cycle. Moreover, significant E3-instanton contributions only arise from instantons wrapped on the small four-cycles. Therefore, we can make the general ansatz for the D-brane and instanton cycles

$$D_{\mathrm{D7}} = n_b\,D_b + n_c\,D_c\,, \qquad D_{\mathrm{E3}} = m_b\,D_b + m_c\,D_c\,, \tag{4.18}$$

where now the wrapping numbers n and m need not be integer. They are related to the wrapping numbers n_i in the $\{\eta_i\}$ basis by

$$n_b = \frac{1}{3}\,n_2\,, \qquad n_c = n_1 - \frac{5}{3}\,n_2\,, \tag{4.19}$$

and similarly for (m_b, m_c).

4.2 Moduli stabilisation

Now that we have collected all the topological data, we can develop our model further. It will turn out that the tadpole cancellation conditions for the present setup impose strong restrictions so that we cannot consider a full MSSM set-up but only a chiral toy model. We will have two stacks of D7-branes wrapping rigid four-cycles D_A and D_B where only on the first one a non-trivial line bundle $\mathcal{L}_A$ is turned on. We consider the Standard Model as being part of the $\mathrm{U}(N_A)$ gauge group on the first stack of branes (even though in the eventual model it will not have large enough gauge group). Then we get MSSM matter from the intersections AA' and AB where the prime denotes the orientifold image. Connecting to our discussion in section 3.3, we in general allow the gauge group $\mathrm{U}(N_A)$ to be larger than just the MSSM gauge group. Then from the two intersections AA' and AB we get matter Φ_{SM} which is part of the Standard Model. Furthermore, we get other matter Φ_{abel} transforming in singlet representations of the MSSM gauge group, but carrying certain charges under abelian $\mathrm{U}(1)$s orthogonal to $\mathrm{U}(1)_Y$. In addition, in order for satisfying the D7-brane tadpoles we need extra hidden sector branes.

Before we give the complete model, let us first elaborate on the D- and F-term constraints.

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D-Term constraints. In section 3.1 we have explained that the D-terms in large volume scenarios should vanish in order not to spoil the $1/\mathcal{V}$ expansion of the scalar F-term potential and the resulting minimum. The D-term contains the Fayet-Iliopoulos parameter ξ and the possible matter fields $\Phi_{\rm SM}$, $\Phi_{\rm exo}$ and $\Phi_{\rm abel}$. However, as argued previously, for the simple reason that the SM gauge symmetry is unbroken at low energies, at least the VEVs of the first two matter fields have to vanish and for $\Phi_{\rm abel}$ it is likely to vanish. For the MSSM sector, we are thus left with the requirement that $\xi_A = \xi_B = 0$. Recalling the precise form of the FI-parameter (3.3), the condition $\xi_A = 0$ reads

$$0 = \int c_1(\mathcal{L}_A) \wedge \left[D_{\rm D7_A}\right] \wedge J \,. \tag{4.20}$$

For the second D7-brane the condition $\xi_B = 0$ is trivially satisfied because of $c_1(\mathcal{L}_B) = 0$. Next, we consider the (chiral) zero mode constraint from the D7-E3 intersections. The only non-trivial equation comes from $\mathrm{D7}_A$ and reads

$$0 = \int c_1(\mathcal{L}_A) \wedge \left[D_{\rm D7_A}\right] \wedge \left[D_{\rm E3}\right] \,. \tag{4.21}$$

Using then our ansatz (4.18) in the diagonal basis, we find that the only suitable solution to the two equations above is

$$J = t_a \left[D_a\right] - t \left[D_{\rm E3}\right] \,. \tag{4.22}$$

Let us note that this solution implies $t_b = \frac{1}{3}\, m_b\, t$ and $t_c = \frac{1}{3}\, m_c\, t$. Comparing with the Kähler cone constraint $t_b > t_c > 0$ and going back to the basis in $\{\eta_1, \eta_2, \eta_3\}$, we see that only wrapping numbers with $2m_2 > m_1 > \frac{5}{3}m_2$ are possible. This cannot be solved by any of the rigid cycles $(m_1, m_2, m_3) \in \{(1,0,0), (1,1,0), (2,1,0), (2,2,0)\}$. However, the choice $(m_1, m_2, m_3) = (2,1,0)$, i.e. $D_{\rm E3} = \frac{1}{3}(D_b + D_c)$, is at least on the boundary of the Kähler cone at $t_b = t_c$. Of course, we cannot choose instantons at will but have to take all of them into account. But we can arrange our setup in such a way that only an instanton along the cycle $(m_1, m_2, m_3) = (2,1,0)$ contributes to the stabilisation of the Kähler moduli. We will come back to this point after we specified the orientifold projection and the D-branes in our model. Note furthermore, by allowing a non-vanishing VEV for $\Phi_{\rm abel}$, it might be possible to fix t_b and t_c on a ray inside the Kähler cone via the instanton above.

Let us now choose the stacks of D7-branes to wrap the rigid four-cycles

$$D_{\rm D7_A} = D_5 + D_6 = \frac{1}{3}\left(D_b - 2\,D_c\right) \,, \qquad D_{\rm D7_B} = D_5 = D_c \,, \tag{4.23}$$

with the line bundles

$$\mathcal{L}_A = \frac{1}{3}(2\,D_b + 5\,D_c) \,, \qquad \mathcal{L}_B = \mathcal{O} \,. \tag{4.24}$$

With this choice, as shown above, there are no chiral zero modes on the $D7 - E3$ intersections. However, similar to [69], we expect both vector-like bosonic and fermionic zero modes, because, as shown in figure 1, the rigid E3-instanton actually contains both $D_{\rm D7_A} = D_5 + D_6$ and $D_{\rm D7_B} = D_5$ as a sub-locus. One way to get rid of these zero modes, would be to turn on discrete Wilson lines or discrete displacement on the D7-brane resp. E3-instanton. It is beyond the scope of this paper to analyse mathematically this possibility for these divisors. From now on, we proceed by assuming that such non-chiral zero modes can be made massive so that indeed the E3-instanton on $D_{\rm E3} = 2D_5 + D_6$ contributes to the uncharged superpotential.

Before concluding this part, let us note that the vanishing of the D-term gives rise to a minimum of the scalar D-term potential. Moreover, we have argued that the F-term potential does not depend on at least one linear combination of Kähler moduli $\overline{\tau}$ which however appears in the D-term. For moduli stabilisation this means that $\partial V/\partial\overline{\tau} = \partial V_D/\partial\overline{\tau} = 0$ is solved by the vanishing D-term and thus in our setup fixes

$$t_b = t_c =: t \,. \tag{4.25}$$

In the diagonal basis this solution implies that D_6 shrinks to zero size but D_5 stays finite. Note first, our Standard Model branes do both involve D_5 and so their volume is always non-zero. Second, for a non-vanishing VEV of $\Phi_{\rm abel}$ we expect the volume of D_6 to be finite.

F-Term constraints. Let us now go on and study the F-term potential. Since we only have a single instanton contributing to the potential, we can refer to equation (3.21). Using then the concrete data of our model, we find $\mathrm{Vol}\big(D_{\rm E3}\cap D_{\rm E3}\big) = -5t_b - t_c$ and therefore

$$V_F \simeq \frac{(2\pi)^2}{\hat{\mathcal{V}}}\,\big(5\hat{t}_b + \hat{t}_c\big)\,\big|A_{\rm E3}\big|^2\, e^{-4\pi\hat{\tau}_{\rm E3}} - \frac{4\pi}{\hat{\mathcal{V}}^2}\,\hat{\tau}_{\rm E3}\, e^{-2\pi\hat{\tau}_{\rm E3}}\,\big|A_{\rm E3}W_0\big| + \frac{3}{4}\,\frac{\hat{\xi}}{\hat{\mathcal{V}}^3}\,\big|W_0\big|^2. \tag{4.26}$$

The first term cannot be expressed as a square root of $\hat{\tau}_{\rm E3} = \frac{1}{6}\big(45\hat{t}_b^2 + 9\hat{t}_c^2\big)$ and so the analysis of [16] for the minimum of V_F at large volumes is not applicable. However, employing equation (4.25), we find the following relation between the volume of the instanton cycle and the volume of its self-intersection

$$\mathrm{Vol}\big(D_{\rm E3}\cap D_{\rm E3}\big) = -6\,\hat{t} = -2\,\sqrt{\hat{\tau}_{\rm E3}}\,. \tag{4.27}$$

Note that this volume formally is negative, which simply reflects the fact that the four-cycle $D_{\rm E3}$ is exceptional with a self-intersection not corresponding to an effective two-cycle. Using this relation, the above expression becomes

$$V_F \simeq \frac{8\pi^2}{\hat{\mathcal{V}}}\,\sqrt{\hat{\tau}_{\rm E3}}\,\big|A_{\rm E3}\big|^2\, e^{-4\pi\hat{\tau}_{\rm E3}} - \frac{4\pi}{\hat{\mathcal{V}}^2}\,\hat{\tau}_{\rm E3}\, e^{-2\pi\hat{\tau}_{\rm E3}}\,\big|A_{\rm E3}W_0\big| + \frac{3}{4}\,\frac{\hat{\xi}}{\hat{\mathcal{V}}^3}\,\big|W_0\big|^2. \tag{4.28}$$

Recalling our discussion in section 3.4, the $1/\hat{\mathcal{V}}$ expansion of the F-term potential is of the form which allows for a minimum of V_F at large values of $\hat{\mathcal{V}}$.

We can then treat these variables as fixed and use their relation to the Kähler moduli. We obtain

$$t_b = t_c = t = \frac{1}{3}\sqrt{\tau_{\rm E3}}\,, \qquad t_a = \left(\frac{6}{5}\,\mathcal{V}_0 + \frac{2}{5}\,\tau_{\rm E3}^{3/2}\right)^{1/3}, \tag{4.29}$$

where we denoted the value of $\mathcal{V}$ in the minimum by $\mathcal{V}_0$. Therefore, in this model all Kähler moduli have been stabilised. To be more precise, we have seen that all coefficients t_a in the expansion of J are fixed and so are the real parts of the Kähler moduli T_i. Furthermore, through the F-term potential the axion corresponding to the instanton cycle is stabilised and via the D-term and Green-Schwarz mechanism the axion associated with the matter sector gets massive.

For the Kähler moduli, we now get three different mass scales. Since the D-term vanishes in the minimum, the mass of the large volume modulus and the small cycle fixed by the instanton do not change. Just keeping track of the $1/\mathcal{V}_0$ factor they scale like $M_{\tau_b} \simeq M_{\rm Pl}/\mathcal{V}_0^{3/2}$ and $M_{\tau_s} \simeq M_{\rm Pl}/\mathcal{V}_0$ [17]. The orthogonal Kähler modulus fixed by the D-term then has mass $M_{\tau_D} \simeq M_{\rm Pl}/\sqrt{\mathcal{V}_0}$, which being of string scale size is much heavier than the other two.

Numerical analysis. In order to explicitly check that the large volume minimum of the full scalar potential persists in our model, we have numerically evaluated equation (4.26). Installing the appropriate factors of 2π and g_s, and choosing $|A_{E3}| = 1$, $|W_0| = 5$, we minimised the function[12]

$$\begin{aligned} V_{F+D}\big(\mathcal{V}, \tau_b, \tau_c\big) = &+ \frac{18.6}{\mathcal{V}} \,\big(\sqrt{5\tau_b} + \sqrt{\tau_c}\,\big)\, g_s\, e^{-\frac{4\pi}{3}\frac{1}{g_s}(\tau_b+\tau_c)} \\ &- \frac{20.9}{\mathcal{V}^2}\,\big(\,\tau_b + \tau_c\,\big)\, g_s^2\, e^{-\frac{2\pi}{3}\frac{1}{g_s}(\tau_b+\tau_c)} + \frac{6.5}{\mathcal{V}^3}\, g_s^3 \\ &+ \frac{13.3}{\mathcal{V}^2}\, \frac{1}{\tau_b - 2\tau_c}\, g_s^3\, \big(\sqrt{5\tau_c} - \sqrt{\tau_b}\,\big)^2 . \end{aligned} \tag{4.30}$$

Note that we have not yet fixed the value of g_s which is determined by the VEV of the dilaton. We have assumed that it is stabilized by fluxes and since we did not perform an explicit analysis of this mechanism, we choose $g_s = 1/10$ for convenience. However, as noted in [18], the stabilised volume $\mathcal{V}$ will depend exponentially on g_s through $\mathcal{V} \sim e^{c/g_s}$ where c is some constant. Thus, a more careful analysis of the flux sector is inevitable.

Coming back to the potential above, we observe that the dominant part of (4.30) is given by the D-term potential fixing the combination $\tau_b = 5\tau_c$. On top of that direction, we found a minimum of the potential in the variables $\mathcal{V}$ and τ_b. In figures 2 and 3, we have plotted two sections through the parameter space showing the potential in the vicinity of the minimum. The numerical values (in string units) in the minimum are $\mathcal{V} \approx 2.2 \cdot 10^{16}$ and the four-cycle volumes are stabilised at $\tau_b \approx 1.63$, $\tau_c \approx 0.33$.[13] For the volume of the Standard Model cycles we find $\tau_{\rm SM} \simeq 0.33$ and the value of the scalar potential in the minimum is of the order $V_{\rm min} \simeq -10^{-54}\, M_{\rm Pl}^4$.

The stabilised four-cycle volumina are in a region where we have to worry whether we can trust the supergravity approximation. Let us investigate more closely what the numerical reason is. Recall from [16] the approximate formulas for the volume $\mathcal{V}$ and the

[12]A very similar potential appeared in [62], but without the D-term part.

[13]If we minimise the potential (3.17) instead of its large volume expansion (4.30), we find the minimum at $\tau_c \simeq 0.53$, $\tau_b \simeq 2.64$ and $\mathcal{V} \simeq 1.1 \cdot 10^{13}$ for $g_s = 1/10$. The difference in the value of $\mathcal{V}$ can be compensated by arranging $g_s = 1/12$ so that the minimum is at $\tau_c \simeq 0.53$, $\tau_b \simeq 2.64$ and $\mathcal{V} \simeq 7 \cdot 10^{15}$.

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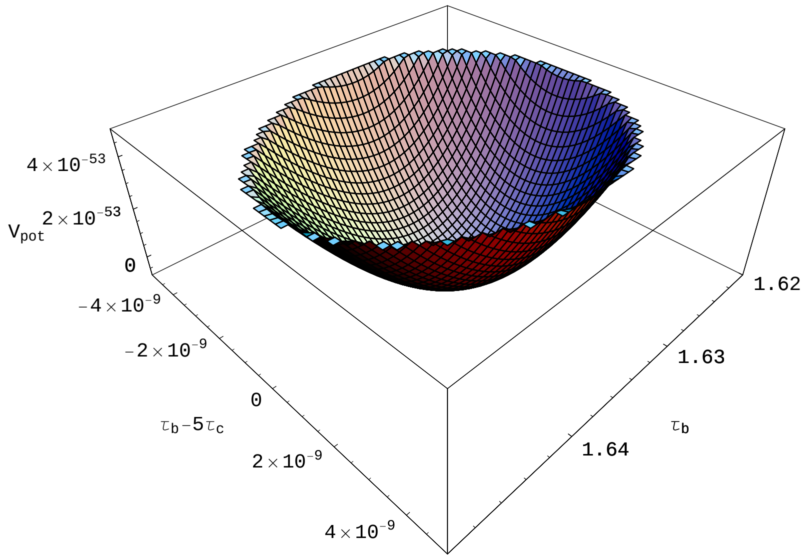

Figure 2: The potential $V(\mathcal{V}, \tau_b, \tau_c)$ for $\mathcal{V} = 2.15 \cdot 10^{16}$.

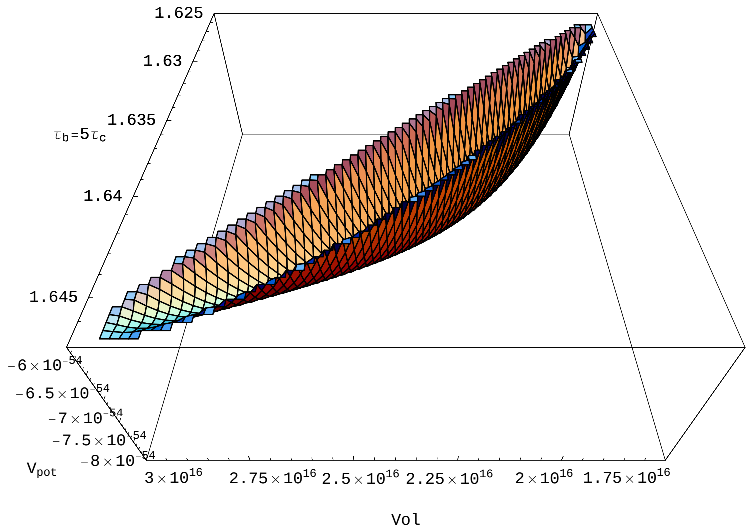

Figure 3: The potential $V(\mathcal{V}, \tau_b, \tau_c)$ for $\tau_c = 0.33$.

four-cycle in the minimum

$$\mathcal{V} \simeq \frac{\mu\, g_s\, |W_0|}{2\,\lambda\, a_s |A_s|} \left(\frac{4\,\nu\lambda\,\xi}{\mu^2}\right)^{1/3} e^{\frac{a_s}{g_s}\left(\frac{4\,\nu\lambda\,\xi}{\mu^2}\right)^{2/3}}\,, \qquad \tau \simeq \left(\frac{4\,\nu\lambda\,\xi}{\mu^2}\right)^{2/3}\,, \tag{4.31}$$

where we have used the notation from equation (2.10). Note that λ contains the information about the intersection of the instanton cycles and thus depends on the topology of the manifold and on the cycles suitable for instantons. Furthermore, ξ is proportional to the Euler characteristic χ and so the above formulas depend strongly on the topology of the compactification manifold.

For our present model, using the data after D-term fixing but leaving the Euler characteristic χ and the string coupling g_s unspecified, we obtain

$$\mathcal{V} \simeq 6.1 \cdot 10^{-2}\, g_s \left(-\chi\right)^{1/3} e^{0.145\,\frac{1}{g_s}\,(-\chi)^{2/3}}\,, \qquad \tau_{\rm SM} \simeq 1.2 \cdot 10^{-2} \left(-\chi\right)^{2/3}. \tag{4.32}$$

Therefore the prefactor of order 10^{-2} in (4.32) and the smallness of the Euler characteristic $\chi = -144$ of our Calabi-Yau manifold are the reasons for the string-frame four-cycle volume $\tau_{\rm SM}$ to come out so small.

Just as a rough estimate, let us analyse for which values of g_s and χ the formulas (4.32) give more realistic values of the Kähler moduli. Choosing for instance $\tau_{\rm SM} = 1.2$ leads to $\chi \simeq -1000$. For the string coupling $g_s = \frac{3}{8}$ we then get $\mathcal{V} = 5 \cdot 10^{15}$. This points towards choosing Calabi-Yau's with Euler-characteristics just at the limit of presently known examples for χ.

4.3 Orientifold with tadpole cancellation

We now show that the setup introduced in the previous section can really be implemented in a globally defined orientifold model. We choose the holomorphic involution σ of the orientifold projection to permute the complex coordinates x_1 and x_2. Then the divisors D_5, D_6, D_7 are invariant and we have $h^+_{1,1} = 3$ and $h^-_{1,1} = 0$. The orientifold plane is given by

$$D_{\rm O7} = 3\eta_1 + 2\eta_2 + \eta_3 = D_a - \frac{1}{3}\, D_b - \frac{1}{3}\, D_c\,, \tag{4.33}$$

and the cohomologies for this cycle are $H^i(D_{\rm O7}, \mathcal{O}) = (1, 0, 3)$. A careful analysis shows that in addition, the orientifold projection also leaves three points on the Calabi-Yau manifold invariant. This can be seen as follows. Choose the intersection of $x_3 = x_4 = x_7 = 0$ which gives five points. These are described as the solutions to the equation $x_1^5 + x_2^5 = 0$ in the variables (x_1, x_2, x_5, x_6) up to the projective identifications shown in (4.2). These latter allow to fix (x_5, x_6) to the point $(1, 1)$ and to see that the solution $(x_1, x_2) = (1, -1)$ is invariant under σ. The other four points are pairwise interchanged. The same story also holds on the intersections $x_3 = x_6 = x_7 = 0$ and $x_3 = x_5 = x_6 = 0$ giving the claimed three O3-planes.

Let us state a criterion by which we can decide whether on a stack of $\Omega\sigma(-1)^{F_L}$ invariant D7-branes we get an SO or SP projection. In the geometric orientifold we are considering, placing the D7-branes right on top of the O7-plane gives an SO(8) gauge symmetry. Wrapping another D7-brane on a σ invariant four-cycle D with trivial gauge bundle, also leads to an SO or SP gauge symmetry. If the configuration is such that D intersects the O7-plane over a two-cycle, then locally around the intersection the open string stretched between the D7-brane on D and the one on $D_{\rm O7}$ has four Neuman-Dirichlet boundary conditions and therefore carries an SP gauge group.

Let us first consider the simple model with eight D7-branes on top of the O7-plane. Using $\int_{D_{\rm O7}} {\rm c}_2(T_{\rm O7}) = 45$ in the eq. (2.15), the resulting D3-brane tadpole is

$$N_{\rm D3} + N_{\rm flux} = 12 \,, \tag{4.34}$$

so that the Euler characteristic of the Calabi-Yau four-fold is $\chi(\mathcal{Y}) = 288$.

Now we are considering the model from the previous section with magnetised D7-branes. For convenience, let us recall its instanton and D7-brane data

$$\begin{aligned}
D_{\rm E3} &= 2\eta_1 + \eta_2 = \tfrac{1}{3}\big(D_b + D_c\big) \,, \qquad & c_1\big(\mathcal{L}_{\rm E3}\big) &= 0 \,, \\
D_A &= \eta_1 + \eta_2 \;\;= \tfrac{1}{3}\big(D_b - 2D_c\big) \,, \qquad & c_1\big(\mathcal{L}_A\big) &= 5\eta_1 + 2\eta_2 = \tfrac{1}{3}\big(2D_b + 5D_c\big) \,, \\
D_B &= \eta_1 \qquad\quad\;\; = D_c \,, \qquad & c_1\big(\mathcal{L}_B\big) &= 0 \,.
\end{aligned} \tag{4.35}$$

Note that since we do not have a gauge bundle on the second D7-brane, it is invariant under the orientifold projection. Because it intersects the O7-plane over a two-cycle, according to our criterion from above it carries an $SP(2N_B)$ gauge symmetry. Similarly, the $\Omega\sigma(-1)^{F_L}$ invariant instanton cycle $D_{\rm E3}$ intersects the O7-plane over a two-cycle and carries therefore an $O(1)$ gauge symmetry.

The chiral matter between the D7-branes can be computed using the rules from table 1. Leaving the number of coincident branes N_A and N_B unspecified, we find

$$10 \times \big[\overline{A}_A\big] + 10 \times \big[\overline{S}_A\big] + 10 \times \big[(N_A, 2N_B)\big] \,. \tag{4.36}$$

However, this spectrum is only free of anomalies if we impose $N_B = N_A$. Thus, the spectrum of our model reads as follows.

$$\begin{array}{|llcl|}
\hline
 & & \mathrm{U}(N_A) \times \mathrm{Sp}(2N_A) & \\
\hline
10 \times (& \overline{\mathbf{A}} & , \quad 1 &) \\
10 \times (& \overline{\mathbf{S}} & , \quad 1 &) \\
10 \times (& \Box & , \quad \Box &) \\
\hline
\end{array} \tag{4.37}$$

Furthermore, we have to satisfy the tadpole cancellation condition for the D7-branes which restricts N_A as $N_A \leq 6$.

The D3-brane tadpole is more involved. The various topological quantities contributing to the formula (2.14) are found as

$$\int_{D_A} {\rm ch}_2\big(\mathcal{L}_A\big) = -5 \,, \quad \int_{D_A} {\rm c}_2\big(T_{D_A}\big) = -17 \,, \quad \int_{D_B} {\rm c}_2\big(T_{D_B}\big) = 3 \,. \tag{4.38}$$

The condition is that, after including also the hidden sector branes, the number of $N_{\rm D3} + N_{\rm flux}$ is non-negative. We found only one solution, which works. We choose the minimal case $N_A = 1$ and in order to satisfy the D7-brane tadpole constraint, we include the hidden branes

$$\begin{aligned}
N_C = 3: &\quad D_C = D_{\rm O7} = 3\eta_1 + 2\eta_2 + \eta_3 \,, \\
N_D = 1: &\quad D_D = \eta_1 + \eta_2 + \eta_3 \,,
\end{aligned} \tag{4.39}$$

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with trivial gauge bundles. The four-cycle D_C is equal to the O7-plane and D_D is a rigid cycle with $H^i(D_D, \mathcal{O}) = (1, 0, 0)$ and $\int_{D_D} \mathrm{c}_2(T_{D_D}) = -37$. Adding up all contributions to the D3-brane tadpole condition gives

$$N_{\mathrm{D3}} + N_{\mathrm{flux}} = 3 \ . \tag{4.40}$$

After having specified the orientifold projection and the D-branes in our model, we can now revisit our claim that only the $O(1)$ instanton along $2D_5 + D_6$ contributes to the stabilisation of the Kähler moduli. Using equation (2.12), we find that there are always chiral zero modes between stringy instantons and D-branes except for $D_{\mathrm{E3}} = 2D_5 + D_6$. Therefore, if present, a term in the superpotential involving the Standard Model fields will be generated. Following our argumentation from section 3.3, such contributions have to be absent because Standard Model fields should not acquire a VEV. Similarly, the contribution from the gauge instanton on top of D_A has to vanish. Thus, only the instanton $D_{\mathrm{E3}} = 2D_5 + D_6$ will contribute to the stabilisation of the Kähler moduli. However, we have to emphasize that actually also the vector-like instanton zero modes have to be determined as well as other mechanism to soak up unwanted fermionic zero modes have to be checked.

To conclude, it is clear that the constraint (4.40) might not give enough freedom for the three-form fluxes to freeze all complex structure moduli. For also including this sector consistently, one needs more involved Calabi-Yau spaces. However, we have demonstrated at a specific swiss-cheese Calabi-Yau manifold with $h_{1,1}(\mathcal{X}) = 3$ that a combination of E3-instantons and D7-brane D-terms can fix all three Kähler moduli in the large volume regime with all small cycles wrapped by D7-branes of order $\log(\mathcal{V})$. In our case, the D-terms (only) fixed the moduli on the boundary of the Kähler cone, where the four-cycle D_6 collapses. Furthermore, we have argued that out of the rigid divisors (4.13) only $D_{\mathrm{E3}} = 2\eta_1 + \eta_2$ contributes to the uncharged superpotential. However, actually the complete vector-like zero mode spectrum has to be computed for such overlapping singular divisors and presumably also discrete Wilson lines and displacements have to be included. This complete mathematical investigation for this specific model is beyond the scope of this paper, whose punchline is rather to exemplify for a concrete Calabi-Yau that the F- and D-term freezing scenario has a good chance to be realisable in concrete large volume Type IIB orientifolds with a chiral D7-brane sector.

5. Conclusions

In this paper we have analysed the problem of combining Kähler moduli stabilisation by instantons resp. gaugino condensation with a chiral D7-brane sector carrying the unbroken chiral gauge theory which we would like to have in four dimensions. Clearly, in order to make progress in deriving viable and predictive string compactifications, this question is of utmost importance.

We argued quite generally, employing both string consistency conditions as well as phenomenological input, that for chiral D7-brane sectors only a combination of F- and D-terms can fix all Kähler moduli. Then we investigated whether for the very promising large volume scenario all their unquestionable nice features can be preserved once these

D-terms are taken into account. We showed that for more than one E3-instanton also the F-term scalar potential contains new terms containing the axionic fields, which potentially destabilise the large volume scenario. Requiring these terms to be absent means that the instanton cycles should not intersect. Moreover, we also allowed for singular four-cycles, which homologically are linear combinations of the elementary ones. These also induce a different moduli dependence in the F-term scalar potential. We plan to investigate the general consequences of such many-instanton contributions in a future work [64].

In this paper we exemplified our general arguments about F-and D-terms by constructing a concrete Type IIB orientifold on a (new) swiss-cheese type Calabi-Yau manifold with three Kähler moduli. Ignoring the details of the three-form flux sector, we constructed a global tadpole cancelling model which showed all the features we do expect for a realistic model. We had a chiral intersecting D7-brane sector and a sector of hidden branes filling up the D7-brane tadpole constraint. Due to chirality there was an induced D-term, fixing (for vanishing VEVs of matter fields) one combination of the Kähler moduli at the boundary of the Kähler cone. We had one rigid small cycle unoccupied by the D7-branes, so that a stringy $O(1)$ E3-instanton wrapped on this cycle contributed to the superpotential. Then the F- and D-terms together fixed the overall volume $\mathcal{V}$ at large values and the two diagonal small ones at size $\tau_i \simeq \log(\mathcal{V})$ in a such a way that another four-cycle collapsed. Of course there will be world-sheet instanton corrections from this collapsed cycle as well as probably also non-negligible string loop corrections, but we only expect them to contribute to the Kähler potential (effectively changing ξ) and to the Fayet-Iliopoulos terms, such that the four-cycle volume is stabilised at order $\tau \simeq \ell_s^4$. Note that, if eventually some of the $\Phi_{\rm abel}$ matter fields are fixed at non-zero value, the D-terms can freeze the Kähler moduli inside the Kähler cone.

We consider our simple toy model as a proof of principle that the LVS can be robust enough that also chiral D7-brane sectors can be introduced. Of course, phenomenologically our model is not satisfying yet. The gauge group and matter content is not realistic and the D3-brane tadpole constraint leaves probably not enough freedom to fix all complex structure moduli by three-form fluxes. Moreover, our analysis of the non-chiral zero modes was not complete. However, we are confident that these shortcoming only reflect the simplicity of the used Calabi-Yau space. Using orientifolds of Calabi-Yau manifolds, for which the D3-brane tadpole is much larger than $\chi(\mathcal{Y}) = 288$ will remedy these problems. To this end, it would be very important to know which of the toric Calabi-Yau manifolds in the list of [70] have a swiss-cheese like structure, respectively can lead to large volume moduli stabilisation. It might be technically very challenging[14] but would be a major step forward to really build completely predictive concrete string compactifications with fluxes and intersecting D7-branes on such more involved Calabi-Yau orientifolds.

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[14] *'Quam multa fieri non posse, priusquam sint facta, iudicantur?'* [Plinius, Naturalis historiae 7, I]

Cohomology	Monomials of degree (m, n, l)
$H^0(\mathcal{M}, \mathcal{L})$	$P(x_1, x_2, x_3, x_4, x_5, x_6, x_7)$
$H^1(\mathcal{M}, \mathcal{L})$	$\frac{P(x_1,x_2,x_3,x_6,x_7)}{x_4 x_5 Q(x_4,x_5)}$ $\frac{P(x_1,x_2,x_3,x_5,x_7)}{x_4 x_6 Q(x_4,x_6)}$ $\frac{P(x_1,x_2,x_3,x_4,x_6)}{x_5 x_7 Q(x_5,x_7)}$ $\frac{P(x_1,x_2,x_3,x_7)}{x_4 x_5 x_6 Q(x_4,x_5,x_6)}$ $\frac{P(x_1,x_2,x_3,x_6)}{x_4 x_5 x_7 Q(x_4,x_5,x_7)}$
$H^2(\mathcal{M}, \mathcal{L})$	0
$H^3(\mathcal{M}, \mathcal{L})$	$\frac{P(x_4,x_5)}{x_1 x_2 x_3 x_6 x_7 Q(x_1,x_2,x_3,x_6,x_7)}$ $\frac{P(x_4,x_6)}{x_1 x_2 x_3 x_5 x_7 Q(x_1,x_2,x_3,x_5,x_7)}$ $\frac{P(x_5,x_7)}{x_1 x_2 x_3 x_4 x_6 Q(x_1,x_2,x_3,x_4,x_6)}$ $\frac{P(x_4,x_5,x_6)}{x_1 x_2 x_3 x_7 Q(x_1,x_2,x_3,x_7)}$ $\frac{P(x_4,x_5,x_7)}{x_1 x_2 x_3 x_6 Q(x_1,x_2,x_3,x_6)}$
$H^4(\mathcal{M}, \mathcal{L})$	$\frac{1}{x_1 x_2 x_3 x_4 x_5 x_6 x_7 Q(x_1,x_2,x_3,x_4,x_5,x_6,x_7)}$

Table 2: Cohomology groups and corresponding monomials for $\mathbb{P}_{[1,3,3,3,5]}[15]$.

knowledge on algebraic geometry with us and to J. Conlon and F. Quevedo for very useful and constructive comments about the issues discussed in this paper.

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A. Cohomology classes of line bundles

In this appendix we combinatorically compute the cohomology classes of general line bundles $\mathcal{L} = \mathcal{O}(m, n, l)$ over the resolution $\mathcal{M}$ of the ambient space $\mathbb{P}_{[1,3,3,3,5]}$. The corresponding classes on the hypersurface $\mathcal{X}$ can then be computed via the Koszul sequence.

$$0 \to \mathcal{L} \otimes \mathcal{O}(-15, -10, -5)_{\mathcal{M}} \to \mathcal{L}_{\mathcal{M}} \to \mathcal{L}_{\mathcal{X}} \to 0 \ . \tag{A.1}$$

Let us recall the resolution

x_1	x_2	x_3	x_4	x_5	x_6	x_7
3	3	3	5	1	0	0
2	2	2	3	0	1	0
1	1	1	1	0	0	1

(A.2)

Then the classes $H^i(\mathcal{M}, \mathcal{L})$ can be computed by counting monomials of degree (m, n, l) [71, 72] as listed in table 2. This can be easily put on a computer. We have checked for many examples that the results are consistent with the Euler characteristic $\chi(\mathcal{X}, \mathcal{L})$ in eq. (4.12).

B. The $\mathbb{P}_{[1,1,3,10,15]}[30]$ Calabi-Yau

Here we will briefly summarise some properties of the Calabi-Yau $\mathbb{P}_{[1,1,3,10,15]}[30]$ as another example of a swiss-cheese like manifold. It has five Kähler moduli out of which four are toric. In the following we collect the toric data for the resolution of the toric singularities.

- The manifold is specified by the resolution

x_1	x_2	x_3	x_4	x_5	x_6	x_7	x_8	p
15	10	3	1	1	0	0	0	30
5	3	1	0	0	1	0	0	10
3	2	0	0	0	0	1	0	6
6	4	1	0	0	0	0	1	12

(B.1)

- The Stanley-Reisner ideal reads

$$SR = \{\, x_2x_7\,, x_2x_8\,,\ x_3x_8\,,\ x_1x_3x_6\,,\ x_1x_6x_7\,,\ x_1x_6x_8\,,\ x_2x_4x_5\,,\ x_3x_4x_5\,,\ x_4x_5x_7\,\}\,. \tag{B.2}$$

- The triple triple intersection numbers in the basis $\eta_1 = D_5$, $\eta_2 = D_6$, $\eta_3 = D_7$, $\eta_4 = D_8$ are encoded in

$$\begin{aligned} I_3 = -\eta_1^3 + 18\eta_2^3 + 8\eta_3^3 + 9\eta_4^3 + 2\eta_1^2\eta_2 + \eta_1^2\eta_4 - 6\eta_1\eta_2^2 \\ - 2\eta_1\eta_3^2 + \eta_3^2\eta_4 - 3\eta_1\eta_4^2 - 3\eta_3\eta_4^2 + \eta_1\eta_3\eta_4\,. \end{aligned} \tag{B.3}$$

- If one expands the Kähler form in the basis $\{\eta_1, \eta_2, \eta_3, \eta_4\}$ as

$$J = t_1[\eta_1] + t_2[\eta_2] + t_3[\eta_3] + t_4[\eta_4]\,, \tag{B.4}$$

then the volumes of the basis divisors are

$$\begin{aligned} \tau_1 &= \frac{1}{2}\Big(-t_1^2 + 4t_1t_2 - 6t_2^2 - 2t_3^2 + 2(t_1+t_3)t_4 - 3t_4^2\Big)\,, \\ \tau_2 &= \Big(t_1 - 3t_2\Big)^2\,, \\ \tau_3 &= \frac{1}{2}\Big(-2t_1 + 4t_3 + 3t_4\Big)\Big(2t_3 - t_4\Big)\,, \\ \tau_4 &= \frac{1}{2}\Big(t_1 + t_3 - 3t_4\Big)^2\,. \end{aligned} \tag{B.5}$$

- The Kähler cone is found by imposing $\int_{\mathcal{C}} J > 0$ which gives the following conditions on the $\{t_i\}$[15]

$$3t_2 + t_3 - 3t_4 > 0\,, \quad t_1 - 3t_2 > 0\,, \quad t_4 - t_3 > 0\,, \quad -t_1 + 2t_2 + t_4 > 0\,. \tag{B.6}$$

[15] We are indebted to Volker Braun for sharing his knowledge and computer program on the computation of the Kähler cone with us.

- Using these restrictions, the overall volume is expressed in terms of the four-cycle volumina as

$$\mathcal{V} = \frac{\sqrt{2}}{45}\left(\left(15\tau_1 + 5\tau_2 + 3\tau_3 + 6\tau_4\right)^{3/2} - \left(3\tau_3 + \tau_4\right)^{3/2} - \frac{5}{\sqrt{2}}\,\tau_2^{3/2} - 5\tau_4^{3/2}\right). \quad \text{(B.7)}$$

  From this we see that by making τ_1 large while keeping the others small, we obtain a swiss-cheese like structure.

- The Euler characteristic χ for the cycle $D = m\,\eta_1 + n\,\eta_2 + p\,\eta_3 + q\,\eta_4$ is

$$\begin{aligned}\chi(\mathcal{X}, \mathcal{O}_D) =& -3mn^2 + \frac{3}{2}q^3 + 3n^3 - \frac{1}{6}m^3 + \frac{1}{2}p^2q + mpq - \frac{3}{2}pq^2 - mp^2 \\ &- \frac{3}{2}mq^2 + \frac{4}{3}p^3 + \frac{1}{2}m^2q + m^2n - n - \frac{1}{3}p - \frac{1}{2}q + \frac{13}{6}m\end{aligned} \quad \text{(B.8)}$$

  where $\mathcal{X}$ stands for $\mathbb{P}_{[1,1,3,10,15]}[30]$. The interesting combinations for the present setup are those with $\chi = 1$ and $m = 0$. Up to wrapping numbers 100, these are

$$(m, n, p, q) = (0, 0, 0, 1)\ ,\quad (0, 0, 1, 0)\ ,\quad (0, 0, 1, 1)\ . \quad \text{(B.9)}$$

- It is more convenient to work in a diagonal basis which we define guided by the form of the volume (B.7)

$$\begin{aligned} D_a &= 15D_1 + 5D_2 + 3D_3 + 6D_4\ , \qquad & D_b &= 3D_3 + D_4\ , \\ D_c &= D_2\ , & D_d &= D_4\ . \end{aligned} \quad \text{(B.10)}$$

  In this basis the total volume reads

$$\mathcal{V} = \frac{\sqrt{2}}{45}\left(\tau_a^{3/2} - \tau_b^{3/2} - \frac{5}{\sqrt{2}}\,\tau_c^{3/2} - 5\tau_d^{3/2}\right)\ , \quad \text{(B.11)}$$

  and the triple intersection numbers again diagonalise

$$I_3 = 225\,D_a^3 + 225\,D_b^3 + 18\,D_c^3 + 9\,D_d^3\ . \quad \text{(B.12)}$$

- Expanding also the Kähler form in this diagonal basis as $J = t_a[D_a] - t_b[D_b] - t_c[D_c] - t_d[D_d]$, we find that the Kähler cone is defined by

$$5\,t_b > t_d > t_c > 0\ , \qquad t_a > t_b + 2\,t_c + t_d\ . \quad \text{(B.13)}$$

- We finally present a list of monomials to be counted in order to determine the cohomology classes $H^i(\mathcal{M}, \mathcal{L})$ on the ambient toric variety. We use the shorthand notation $(1, 2, 4, 5, 7, 8|3, 6)$ for all monomials of the form $\frac{P(x_1,x_2,x_4,x_5,x_7,x_8)}{x_3x_6Q(x_3,x_6)}$ and similarly for the others.

Cohomology	Monomials of degree (m,n,p,q)
$H^0(\mathcal{M},\mathcal{L})$	$(1,2,3,4,5,6,7,8\vert)$
$H^1(\mathcal{M},\mathcal{L})$	$(1,2,4,5,7,8\vert 3,6)$ $(1,2,4,5,6,8\vert 3,7)$ $(1,2,3,4,5,7\vert 6,8)$ $(1,2,4,5,8\vert 3,6,7)$ $(1,2,4,5,7\vert 3,6,8)$
$H^2(\mathcal{M},\mathcal{L})$	$(3,4,5,6,8\vert 1,2,7)$ $(3,4,5,6,7\vert 1,2,8)$ $(2,3,4,5,8\vert 1,6,7)$ $(1,3,6,7,8\vert 2,4,5)$ $(1,2,6,7,8\vert 3,4,5)$ $(1,2,3,6,7\vert 4,5,8)$ $(1,2,7,8\vert 3,4,5,6)$ $(1,2,6,7\vert 3,4,5,8)$ $(2,3,4,5\vert 1,6,7,8)$ $(2,4,5,8\vert 1,3,6,7)$ $(1,2,3,7\vert 4,5,6,8)$ $(1,2,6,8\vert 3,4,5,7)$ $(3,4,5,6\vert 1,2,7,8)$ $(3,4,5,8\vert 1,2,6,7)$ $(1,6,7,8\vert 2,3,4,5)$ $(1,3,6,7\vert 2,4,5,8)$ $(4,5,6,8\vert 1,2,3,7)$ $(3,4,5,7\vert 1,2,6,8)$ $(1,2,7\vert 3,4,5,6,8)$ $(1,2,8\vert 3,4,5,6,7)$ $(1,6,7\vert 2,3,4,5,8)$ $(2,4,5\vert 1,3,6,7,8)$ $(3,4,5\vert 1,2,6,7,8)$ $(4,5,8\vert 1,2,3,6,7)$
$H^3(\mathcal{M},\mathcal{L})$	$(3,6\vert 1,2,4,5,7,8)$ $(3,7\vert 1,2,4,5,6,8)$ $(6,8\vert 1,2,3,4,5,7)$ $(3,6,7\vert 1,2,4,5,8)$ $(3,6,8\vert 1,2,4,5,7)$
$H^4(\mathcal{M},\mathcal{L})$	$(\vert 1,2,3,4,5,6,7,8)$

Table 3: Cohomology groups and corresponding monomials for $\mathbb{P}_{[1,1,3,10,15]}[30]$.

References

[1] A.M. Uranga, *Chiral four-dimensional string compactifications with intersecting D-branes*, *Class. and Quant. Grav.* **20** (2003) S373 [hep-th/0301032].

[2] F.G. Marchesano Buznego, *Intersecting D-brane models*, hep-th/0307252.

[3] D. Lüst, *Intersecting brane worlds: a path to the standard model?*, *Class. and Quant. Grav.* **21** (2004) S1399 [hep-th/0401156].

[4] R. Blumenhagen, M. Cvetič, P. Langacker and G. Shiu, *Toward realistic intersecting D-brane models*, *Ann. Rev. Nucl. Part. Sci.* **55** (2005) 71 [hep-th/0502005].

[5] M. Graña, *Flux compactifications in string theory: a comprehensive review*, *Phys. Rept.* **423** (2006) 91 [hep-th/0509003].

[6] M.R. Douglas and S. Kachru, *Flux compactification*, *Rev. Mod. Phys.* **79** (2007) 733 [hep-th/0610102].

[7] R. Blumenhagen, B. Körs, D. Lüst and S. Stieberger, *Four-dimensional string compactifications with D-branes, orientifolds and fluxes*, *Phys. Rept.* **445** (2007) 1 [hep-th/0610327].

[8] F. Denef, M.R. Douglas and S. Kachru, *Physics of string flux compactifications*, *Ann. Rev. Nucl. Part. Sci.* **57** (2007) 119 [hep-th/0701050].

[9] S.B. Giddings, S. Kachru and J. Polchinski, *Hierarchies from fluxes in string compactifications*, *Phys. Rev.* **D 66** (2002) 106006 [hep-th/0105097].

[10] S. Kachru, R. Kallosh, A. Linde and S.P. Trivedi, *De Sitter vacua in string theory*, *Phys. Rev.* **D 68** (2003) 046005 [hep-th/0301240].

[11] F. Denef, M.R. Douglas and B. Florea, *Building a better racetrack*, *JHEP* **06** (2004) 034 [hep-th/0404257].

[12] D. Robbins and S. Sethi, *A barren landscape*, *Phys. Rev.* **D 71** (2005) 046008 [hep-th/0405011].

[13] F. Denef, M.R. Douglas, B. Florea, A. Grassi and S. Kachru, *Fixing all moduli in a simple F-theory compactification*, *Adv. Theor. Math. Phys.* **9** (2005) 861 [hep-th/0503124].

[14] D. Lüst, S. Reffert, W. Schulgin and S. Stieberger, *Moduli stabilization in type IIB orientifolds. I: orbifold limits*, *Nucl. Phys.* **B 766** (2007) 68 [hep-th/0506090].

[15] D. Lüst, S. Reffert, E. Scheidegger, W. Schulgin and S. Stieberger, *Moduli stabilization in type IIB orientifolds. II*, *Nucl. Phys.* **B 766** (2007) 178 [hep-th/0609013].

[16] V. Balasubramanian, P. Berglund, J.P. Conlon and F. Quevedo, *Systematics of moduli stabilisation in Calabi-Yau flux compactifications*, *JHEP* **03** (2005) 007 [hep-th/0502058].

[17] J.P. Conlon, F. Quevedo and K. Suruliz, *Large-volume flux compactifications: moduli spectrum and D3/D7 soft supersymmetry breaking*, *JHEP* **08** (2005) 007 [hep-th/0505076].

[18] J.P. Conlon, S.S. AbdusSalam, F. Quevedo and K. Suruliz, *Soft SUSY breaking terms for chiral matter in IIB string compactifications*, *JHEP* **01** (2007) 032 [hep-th/0610129].

[19] J.P. Conlon, C.H. Kom, K. Suruliz, B.C. Allanach and F. Quevedo, *Sparticle spectra and LHC signatures for large volume string compactifications*, *JHEP* **08** (2007) 061 [arXiv:0704.3403].

[20] K. Choi, A. Falkowski, H.P. Nilles and M. Olechowski, *Soft supersymmetry breaking in KKLT flux compactification*, *Nucl. Phys.* **B 718** (2005) 113 [hep-th/0503216].

[21] A. Achucarro, B. de Carlos, J.A. Casas and L. Doplicher, *De Sitter vacua from uplifting D-terms in effective supergravities from realistic strings*, *JHEP* **06** (2006) 014 [hep-th/0601190].

[22] O. Lebedev, H.P. Nilles and M. Ratz, *De Sitter vacua from matter superpotentials*, *Phys. Lett.* **B 636** (2006) 126 [hep-th/0603047].

[23] G. Villadoro and F. Zwirner, *D terms from D-branes, gauge invariance and moduli stabilization in flux compactifications*, *JHEP* **03** (2006) 087 [hep-th/0602120].

[24] D. Cremades, M.P. Garcia del Moral, F. Quevedo and K. Suruliz, *Moduli stabilisation and de Sitter string vacua from magnetised D7 branes*, *JHEP* **05** (2007) 100 [hep-th/0701154].

[25] S.L. Parameswaran and A. Westphal, *De Sitter string vacua from perturbative Kähler corrections and consistent D-terms*, *JHEP* **10** (2006) 079 [hep-th/0602253].

[26] S. Gukov, C. Vafa and E. Witten, *CFT's from Calabi-Yau four-folds*, *Nucl. Phys.* **B 584** (2000) 69 [hep-th/9906070].

[27] K. Becker, M. Becker, M. Haack and J. Louis, *Supersymmetry breaking and α'-corrections to flux induced potentials*, *JHEP* **06** (2002) 060 [hep-th/0204254].

JHEP01(2008)058

[28] E. Witten, *Non-perturbative superpotentials in string theory*, *Nucl. Phys.* **B 474** (1996) 343 [hep-th/9604030].

[29] J.P. Conlon, *Moduli stabilisation and applications in IIB string theory*, *Fortschr. Phys.* **55** (2007) 287 [hep-th/0611039].

[30] M. Berg, M. Haack and E. Pajer, *Jumping through loops: on soft terms from large volume compactifications*, *JHEP* **09** (2007) 031 [arXiv:0704.0737].

[31] G. Curio and V. Spillner, *On the modified KKLT procedure: a case study for the $P(11169)(18)$ model*, *Int. J. Mod. Phys.* **A 22** (2007) 3463 [hep-th/0606047].

[32] A. Misra and P. Shukla, *Area codes, large volume (non-)perturbative α'- and instanton-corrected non-supersymmetric (A)dS minimum, the inverse problem and fake superpotentials for multiple-singular-loci-two-parameter Calabi-Yau's*, arXiv:0707.0105.

[33] J.P. Conlon and F. Quevedo, *Gaugino and scalar masses in the landscape*, *JHEP* **06** (2006) 029 [hep-th/0605141].

[34] J.P. Conlon and D. Cremades, *The neutrino suppression scale from large volumes*, *Phys. Rev. Lett.* **99** (2007) 041803 [hep-ph/0611144].

[35] M. Haack, D. Krefl, D. Lüst, A. Van Proeyen and M. Zagermann, *Gaugino condensates and D-terms from D7-branes*, *JHEP* **01** (2007) 078 [hep-th/0609211].

[36] M. Cicoli, J.P. Conlon and F. Quevedo, *Systematics of string loop corrections in type IIB Calabi-Yau flux compactifications*, arXiv:0708.1873.

[37] A. Brignole, L.E. Ibáñez and C. Muñoz, *Soft supersymmetry-breaking terms from supergravity and superstring models*, hep-ph/9707209.

[38] D.S. Freed and E. Witten, *Anomalies in string theory with D-branes*, hep-th/9907189.

[39] G. Villadoro and F. Zwirner, *De Sitter vacua via consistent D-terms*, *Phys. Rev. Lett.* **95** (2005) 231602 [hep-th/0508167].

[40] P.S. Aspinwall, *D-branes on Calabi-Yau manifolds*, hep-th/0403166.

[41] M.R. Douglas and W. Taylor, *The landscape of intersecting brane models*, *JHEP* **01** (2007) 031 [hep-th/0606109].

[42] S. Sethi, C. Vafa and E. Witten, *Constraints on low-dimensional string compactifications*, *Nucl. Phys.* **B 480** (1996) 213 [hep-th/9606122].

[43] C.P. Burgess, R. Kallosh and F. Quevedo, *De Sitter string vacua from supersymmetric D-terms*, *JHEP* **10** (2003) 056 [hep-th/0309187].

[44] E. Dudas and S.K. Vempati, *Large D-terms, hierarchical soft spectra and moduli stabilisation*, *Nucl. Phys.* **B 727** (2005) 139 [hep-th/0506172].

[45] M. Billó et al., *Classical gauge instantons from open strings*, *JHEP* **02** (2003) 045 [hep-th/0211250].

[46] R. Blumenhagen, M. Cvetič and T. Weigand, *Spacetime instanton corrections in 4D string vacua — The seesaw mechanism for D-brane models*, *Nucl. Phys.* **B 771** (2007) 113 [hep-th/0609191].

[47] L.E. Ibáñez and A.M. Uranga, *Neutrino Majorana masses from string theory instanton effects*, *JHEP* **03** (2007) 052 [hep-th/0609213].

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[48] M. Bianchi and E. Kiritsis, *Non-perturbative and flux superpotentials for type-I strings on the Z_3 orbifold*, *Nucl. Phys.* **B 782** (2007) 26 [hep-th/0702015].

[49] N. Akerblom, R. Blumenhagen, D. Lüst and M. Schmidt-Sommerfeld, *Instantons and holomorphic couplings in intersecting D-brane models*, *JHEP* **08** (2007) 044 [arXiv:0705.2366].

[50] T.W. Grimm, *Non-perturbative corrections and modularity in $N = 1$ type IIB compactifications*, *JHEP* **10** (2007) 004 [arXiv:0705.3253].

[51] R. Blumenhagen, M. Cvetič, R. Richter and T. Weigand, *Lifting D-instanton zero modes by recombination and background fluxes*, *JHEP* **10** (2007) 098 [arXiv:0708.0403].

[52] M. Cvetič and T. Weigand, *Hierarchies from D-brane instantons in globally defined Calabi-Yau orientifolds*, arXiv:0711.0209.

[53] R. Argurio, M. Bertolini, G. Ferretti, A. Lerda and C. Petersson, *Stringy instantons at orbifold singularities*, *JHEP* **06** (2007) 067 [arXiv:0704.0262].

[54] M. Bianchi, F. Fucito and J.F. Morales, *D-brane instantons on the T^6/Z_3 orientifold*, *JHEP* **07** (2007) 038 [arXiv:0704.0784].

[55] L.E. Ibáñez, A.N. Schellekens and A.M. Uranga, *Instanton induced neutrino Majorana masses in CFT orientifolds with MSSM-like spectra*, *JHEP* **06** (2007) 011 [arXiv:0704.1079].

[56] I. Garcia-Etxebarria and A.M. Uranga, *Non-perturbative superpotentials across lines of marginal stability*, arXiv:0711.1430.

[57] L.E. Ibáñez and A.M. Uranga, *Instanton induced open string superpotentials and branes at singularities*, arXiv:0711.1316.

[58] C. Petersson, *Superpotentials from stringy instantons without orientifolds*, arXiv:0711.1837.

[59] N. Akerblom, R. Blumenhagen, D. Lüst, E. Plauschinn and M. Schmidt-Sommerfeld, *Non-perturbative SQCD superpotentials from string instantons*, *JHEP* **04** (2007) 076 [hep-th/0612132].

[60] M.P. Garcia del Moral, *A new mechanism of Kähler moduli stabilization in type IIB theory*, *JHEP* **04** (2006) 022 [hep-th/0506116].

[61] T. Maillard, *Toward metastable string vacua from magnetized branes*, arXiv:0708.0823.

[62] J.P. Conlon, *The QCD axion and moduli stabilisation*, *JHEP* **05** (2006) 078 [hep-th/0602233].

[63] K. Bobkov, *Volume stabilization via α' corrections in type IIB theory with fluxes*, *JHEP* **05** (2005) 010 [hep-th/0412239].

[64] R. Blumenhagen, S. Moster and E. Plauschinn, work in progress.

[65] A.-K. Kashani-Poor and A. Tomasiello, *A stringy test of flux-induced isometry gauging*, *Nucl. Phys.* **B 728** (2005) 135 [hep-th/0505208].

[66] V. Braun, M. Kreuzer, B.A. Ovrut and E. Scheidegger, *Worldsheet instantons, torsion curves, and non-perturbative superpotentials*, *Phys. Lett.* **B 649** (2007) 334 [hep-th/0703134].

[67] V. Braun, M. Kreuzer, B.A. Ovrut and E. Scheidegger, *Worldsheet instantons and torsion curves. Part A: direct computation*, *JHEP* **10** (2007) 022 [hep-th/0703182].

JHEP01(2008)058

[68] V. Braun, M. Kreuzer, B.A. Ovrut and E. Scheidegger, *Worldsheet instantons and torsion curves. Part B: mirror symmetry*, *JHEP* **10** (2007) 023 [arXiv:0704.0449].

[69] B. Florea, S. Kachru, J. McGreevy and N. Saulina, *Stringy instantons and quiver gauge theories*, *JHEP* **05** (2007) 024 [hep-th/0610003].

[70] M. Kreuzer and H. Skarke, *Complete classification of reflexive polyhedra in four dimensions*, *Adv. Theor. Math. Phys.* **4** (2002) 1209 [hep-th/0002240].

[71] J. Distler, B.R. Greene and D.R. Morrison, *Resolving singularities in* $(0,2)$ *models*, *Nucl. Phys.* **B 481** (1996) 289 [hep-th/9605222].

[72] R. Blumenhagen, *Target space duality for* $(0,2)$ *compactifications*, *Nucl. Phys.* **B 513** (1998) 573 [hep-th/9707198].

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